I. Turbulence and II. Shocks

GISM 2023, Banyuls

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Outline part I: TURBULENCE

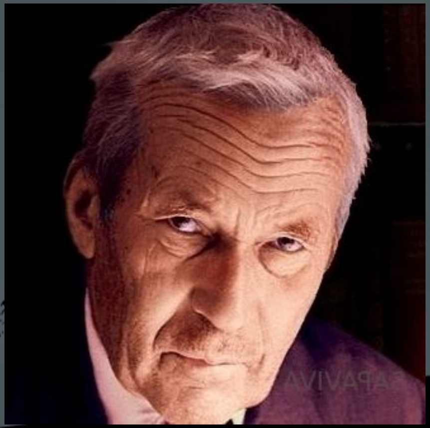
- Introduction
- Reynolds number
- The energy cascade and Kolmogorov (1941)
- Kolmogorov (1962)
- Intermittency
- More advanced topics (Compressible, MHD, HKM)
- Bibliography

What is turbulence ?

Some known statistical properties of 3D *incompressible homogeneous* turbulence

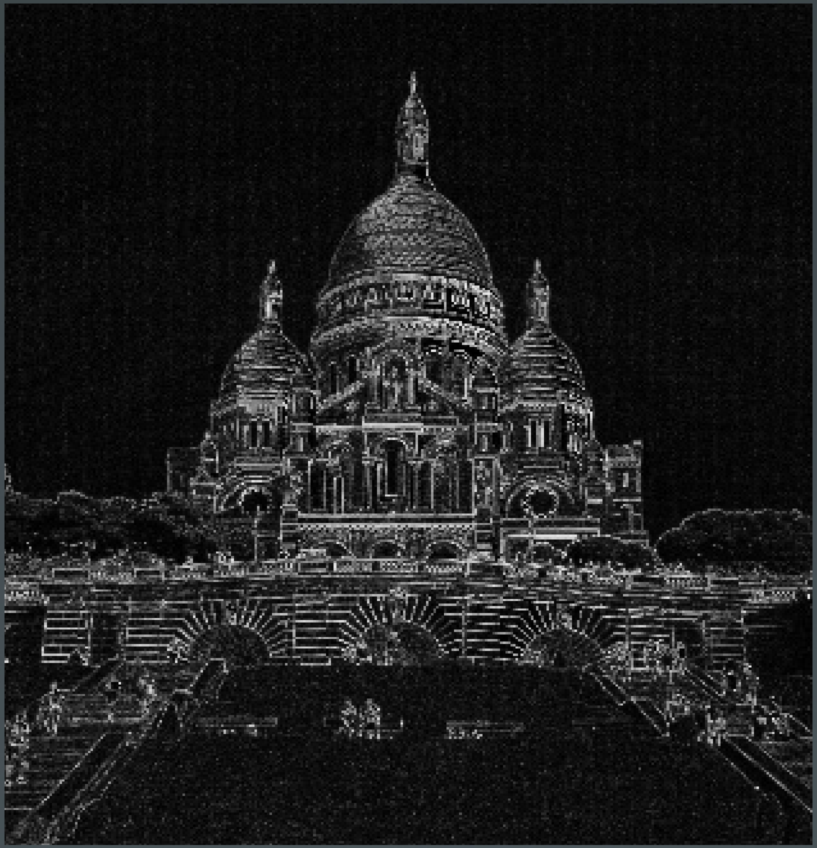
- Kolmogorov (1941a) : ***power spectrum*** $E_u(k) \sim k^{-5/3}$
- Howarth-Karman-Monin equation → ***energy transfer*** function $< (\delta_\ell u_{//})^3 > = - 4/5 <\varepsilon> \ell$ for ℓ in the inertial range. Known as the "4/5th law".
- Kolmogorov (1962) : ***intermittency*** $P(\log \varepsilon) \sim$ Gaussian

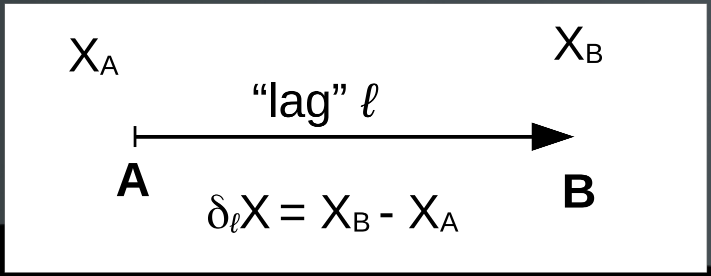
→ lots of measurements and theories on the statistics of increments $\delta_\ell F = F(x + \ell) - F(x)$



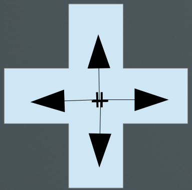
Andreï Nikolaïevitch Kolmogorov's Legacy

Preliminary: increments





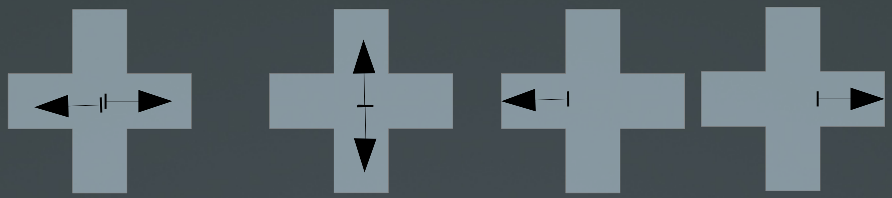
Increments with 1-pixel lags



(Standard deviation of 1-pixel neighbours differences)

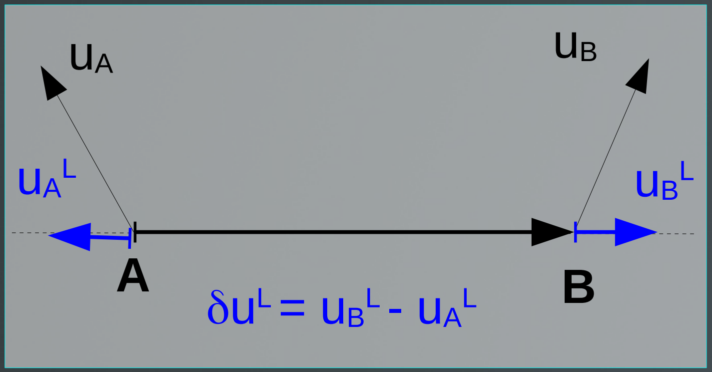
Various types of increments

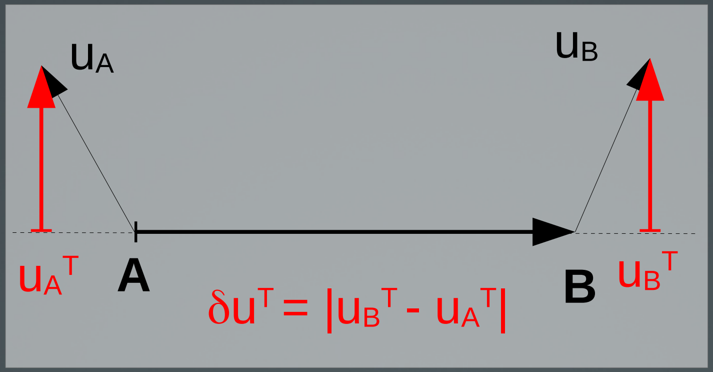
- Directional increments



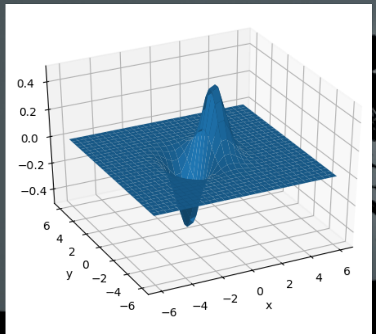
- Increments of vectors need to define a direction:

longitudinal and ***transverse*** increments





- Norm of smoothed gradients
- Convolution with a wavelet



Reynolds number

- Incompressible Navier-Stokes equation

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\mathbf{u} = -\frac{1}{\rho}\nabla p + \nu \nabla^2 \mathbf{u}, \qquad \nabla \cdot \mathbf{u} = 0.$$

- ΔU, L typical velocity and length scales
- ν viscosity

→ Only one dimensionless number: Re=ΔU.L/ν

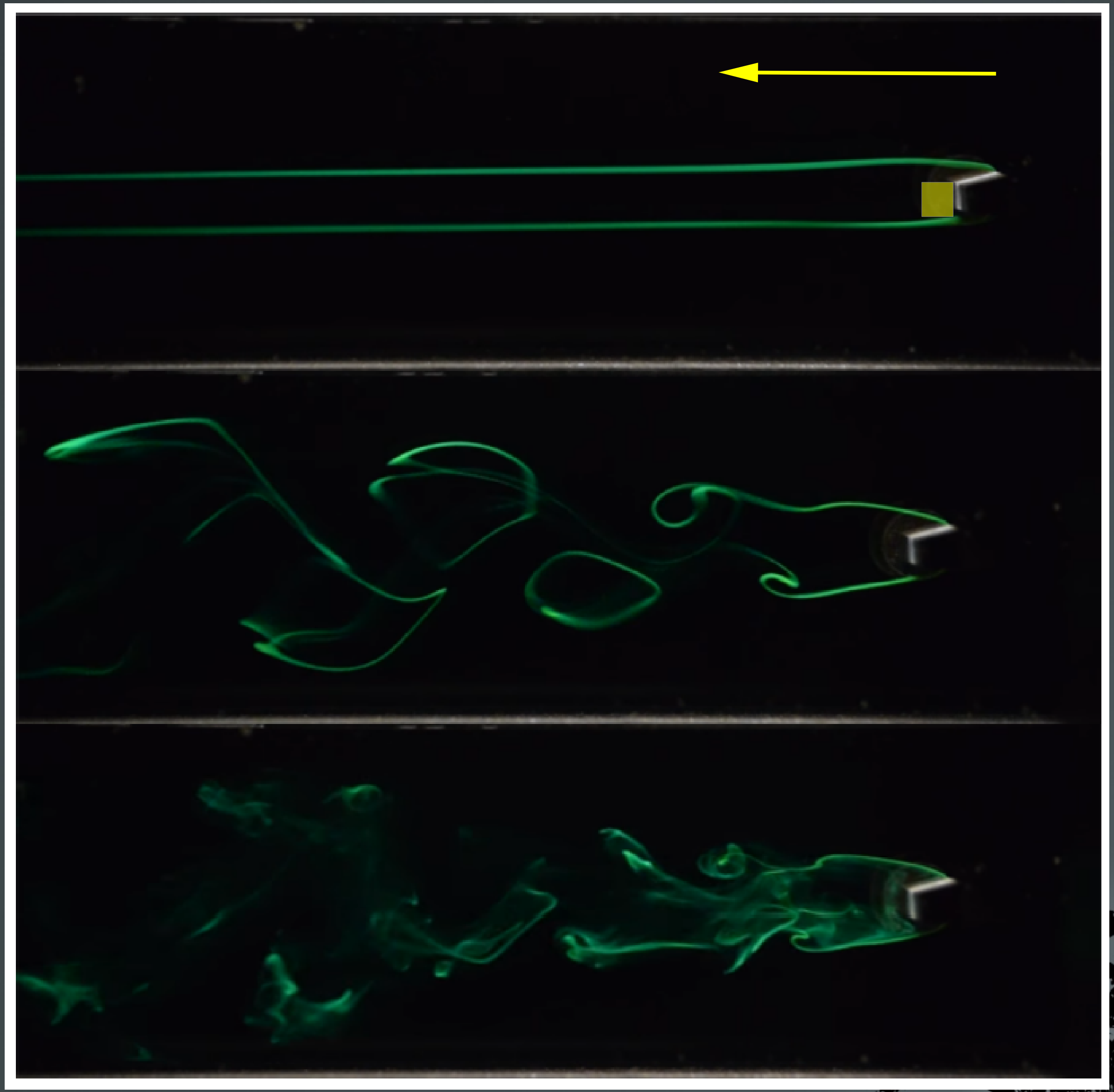
$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = - \nabla p + \frac{1}{\mathrm{Re}} \nabla^2 \mathbf{u}$$

Weak and developped turbulence

Laminar (Re~1) regime

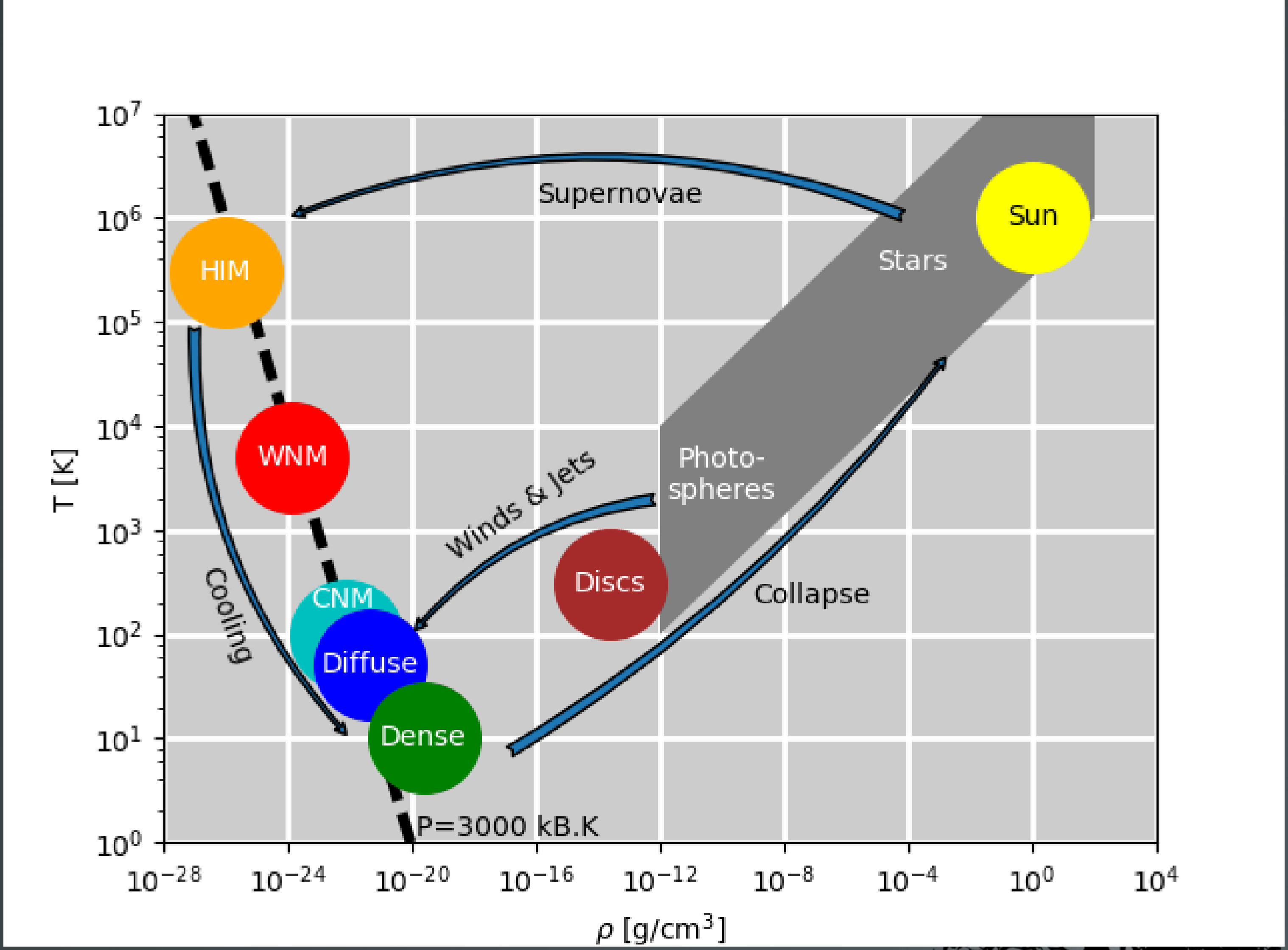
Weak turbulence (Re~100)

Developped turbulence
(Re>1000)



Images from
“le projet Lutécium”, PSL
https://www.youtube.com/watch?v=eD7LdS6bfOQ

Interstellar matter & its cycle



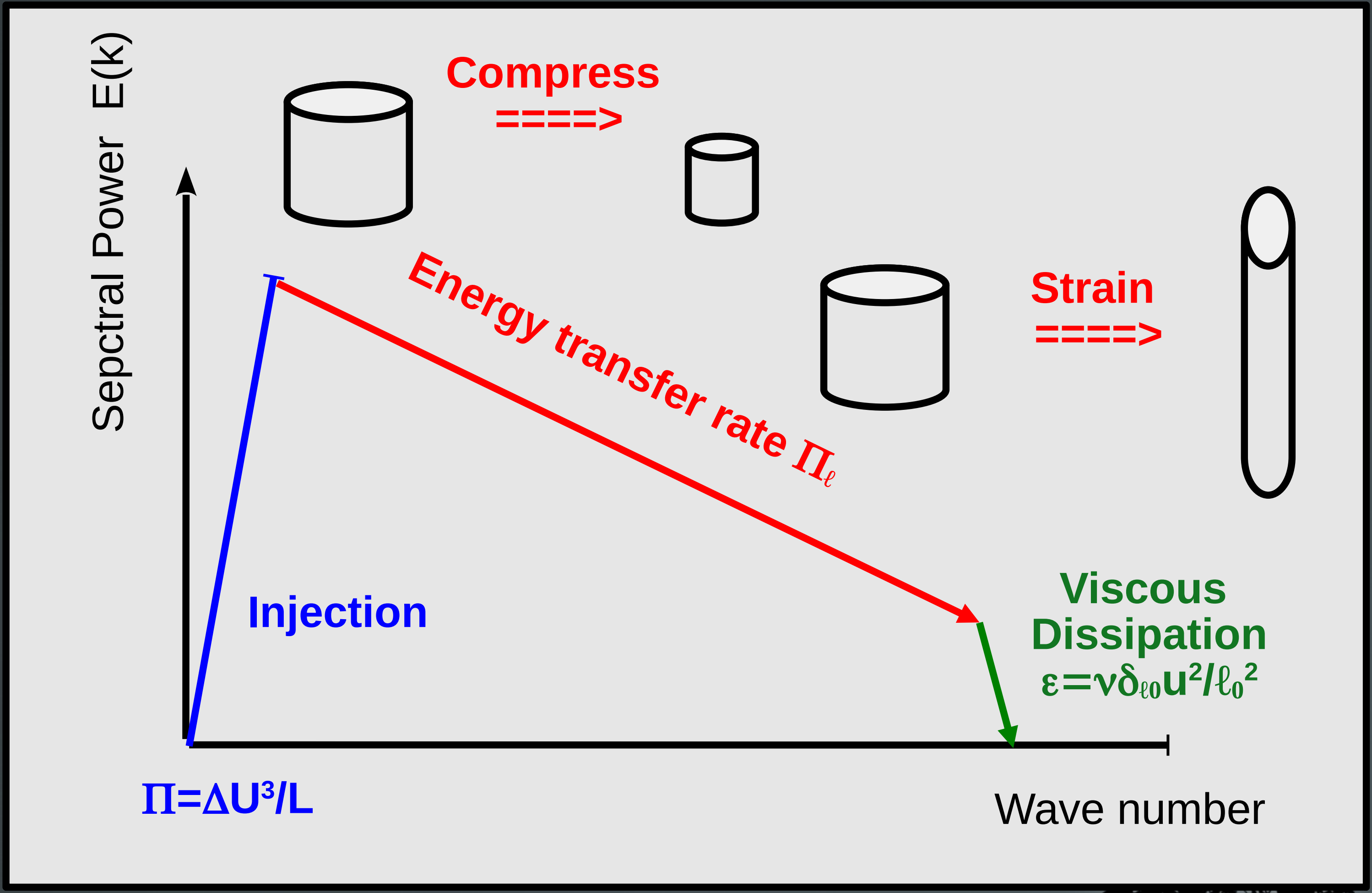
Dimension(less) numbers in our galaxy

	HIM	WNM	CNM	Diffuse	Dense	Discs	Sun
Density ρ [cm^{-3}]	0.004	0.6	30	200	10^4	10^{10}	1 $g.cm^{-3}$
Temperature T [K]	3.10^5	5000	100	50	10	300	10^6
Length scale L [pc]	100	50	10	3	0.1	200 AU	5.10^{-3} AU
Velocity U [$km.s^{-1}$]	10	10	10	3	0.1	0.1	1
$\mathcal{M}$	0.2	2	13	7	0.5	0.1	0.02
$\mathcal{M}_\mathcal{G}$	130	20	15	6	0.8	0.08	0.003
Reynolds : $\mathcal{R}$	10^2	10^5	10^7	10^7	10^6	10^9	10^{17}
$\mathcal{R}_m$	10^{21}	10^{20}	10^{18}	10^{17}	10^{15}	10^9	10^{10}
$\mathcal{R}_{AD}$	10^3	10^3	10^2	10^3	10^4	10^5	10^{20}
Ionisation fraction	1	10^{-2}	10^{-4}	10^{-4}	10^{-4}	10^{-7}	1
Mass per ion [amu]	1	1	12	12	12	24	1
$N_e B_z$ [$3.10^{18} cm^{-2} \mu G$]	1.2	0.9	0.09	0.1	0.2	2	3×10^{16}
N_e[$10^{18} cm^{-2}$]	0.5	5	3	3	1	10^4	10^{28}

Values from Draine's book

(HDR P. Lesaffre)

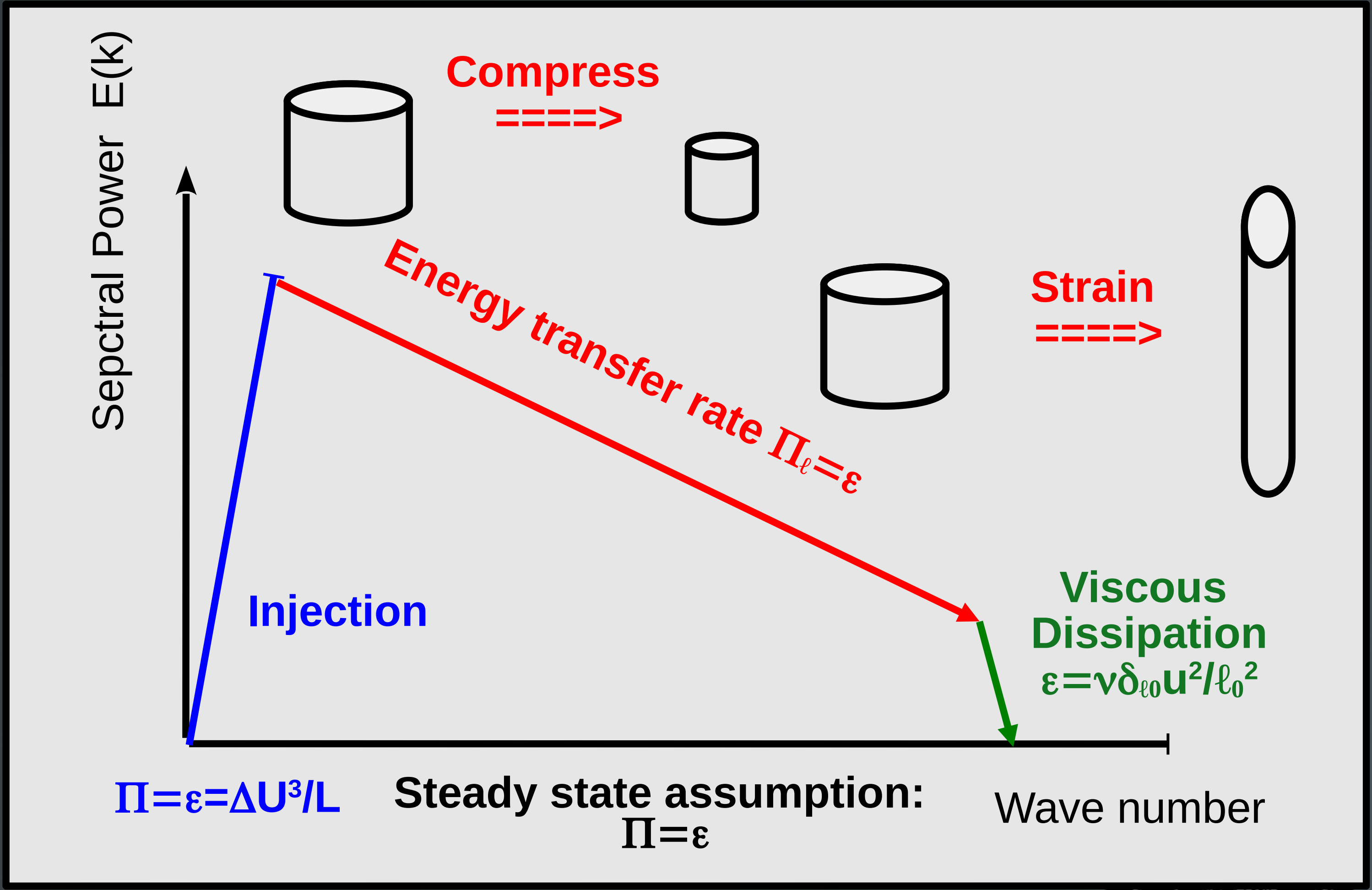
The Kolmogorov cascade and the energy spectrum



Large scales

Small scales

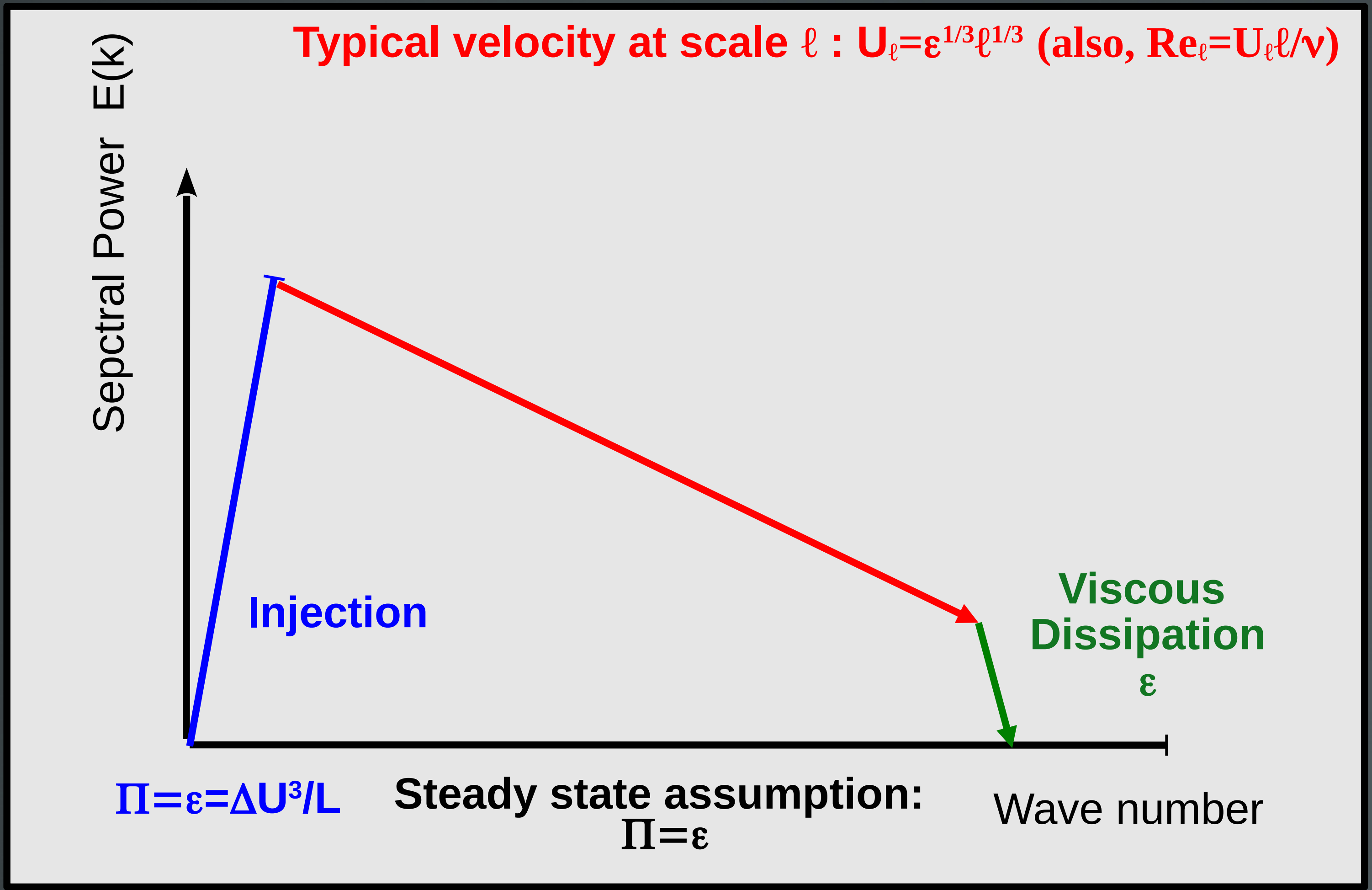
The Kolmogorov cascade and the energy spectrum



Large scales

Small scales

Kolmogorov (1941a) *[light version...]*



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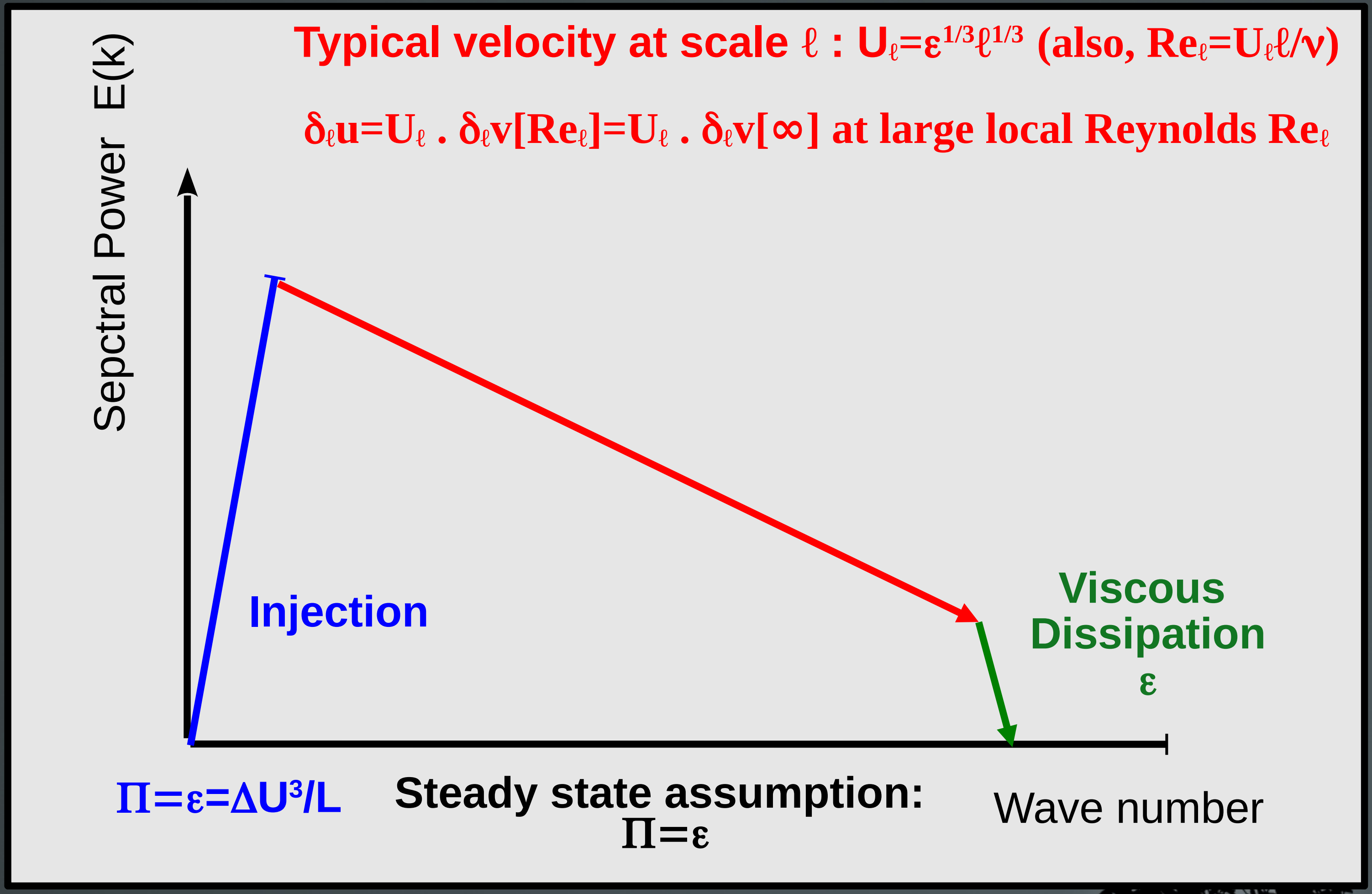

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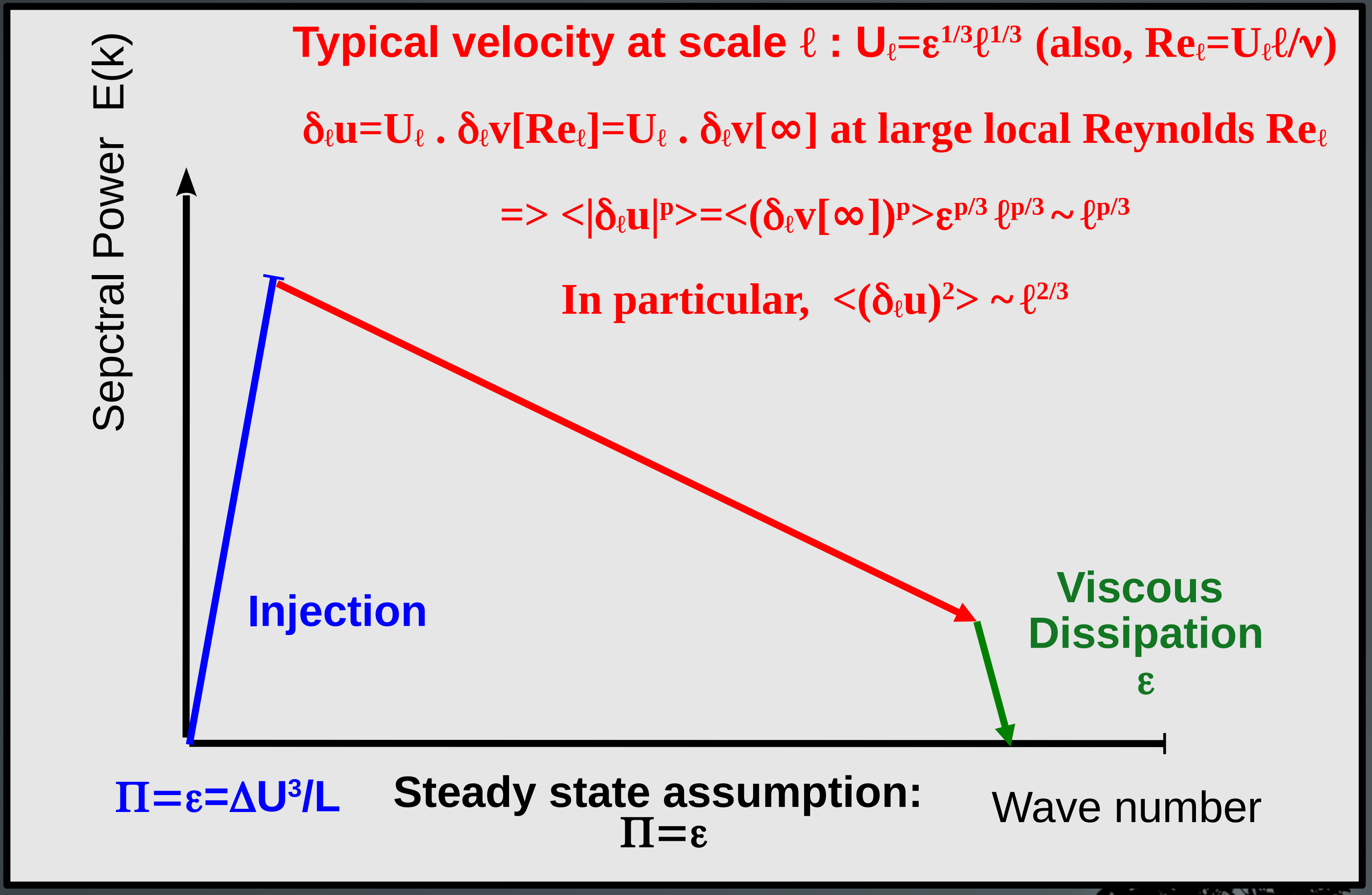

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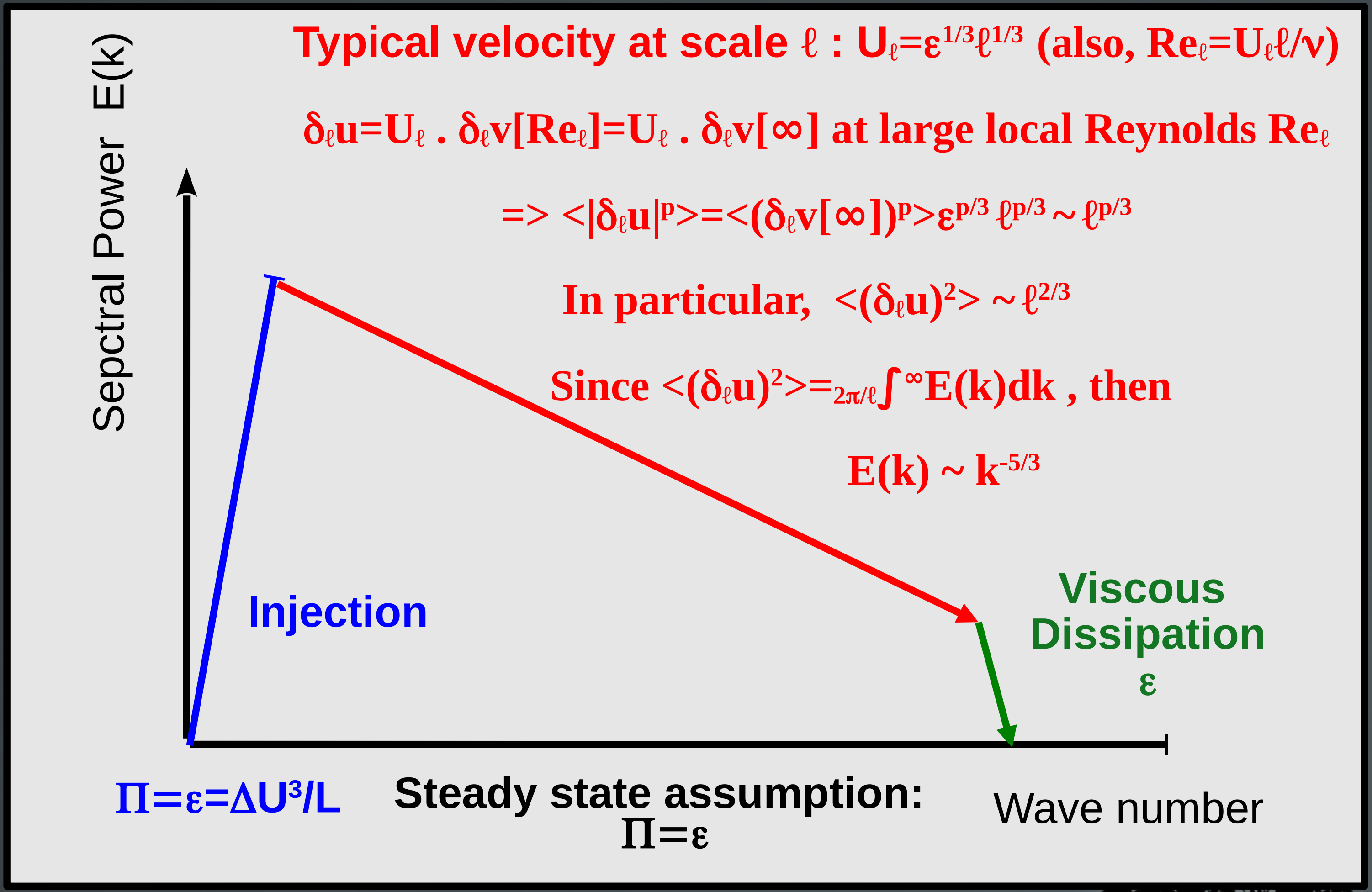


Large scales

Small scales

K41 assumptions and Kolmogorov dissipation scale

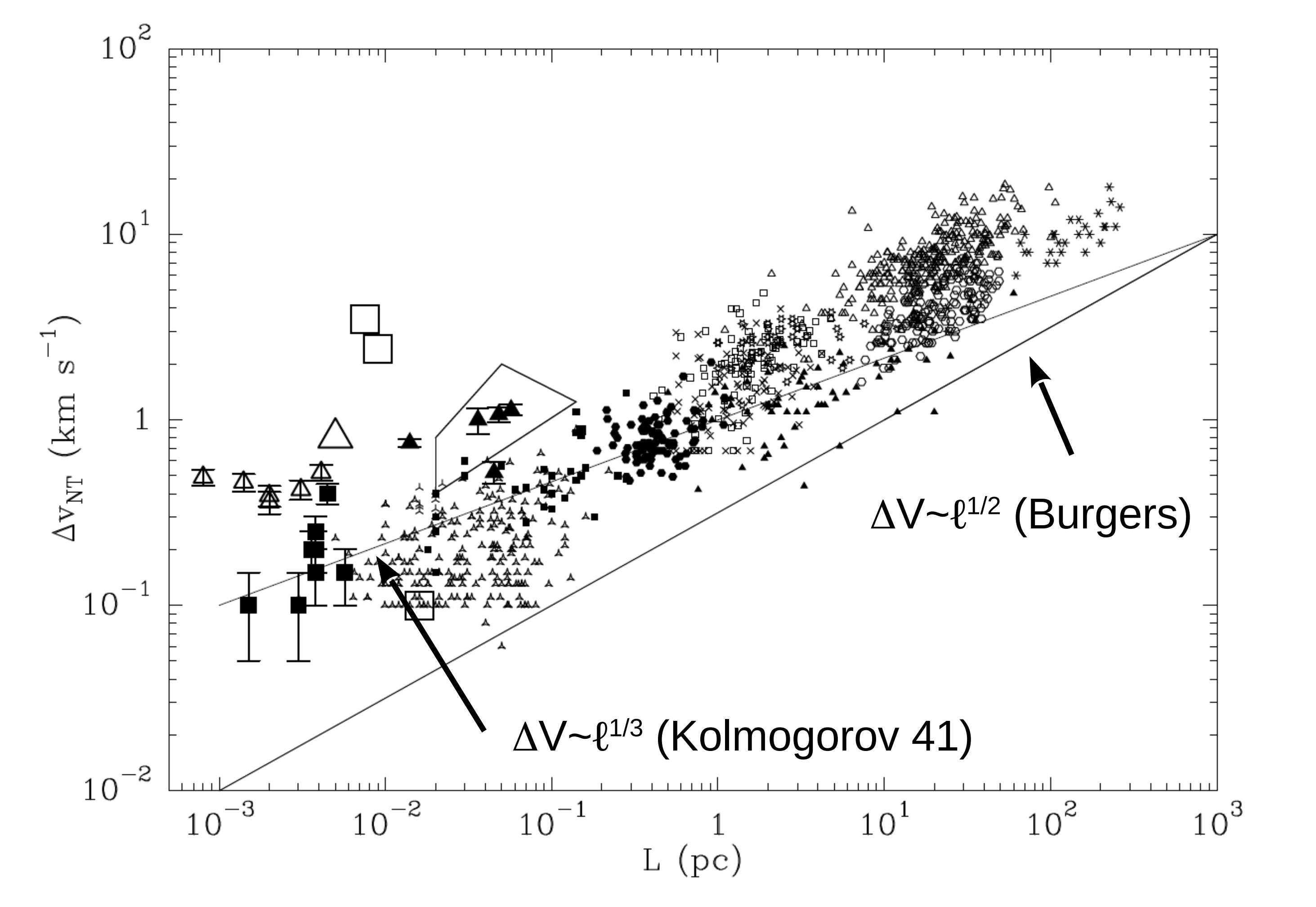
- Steady state assumption: $\Pi = \Pi_\ell = \varepsilon$
- Velocity increments depend on local Reynolds Re_ℓ
- Homogeneous and isotropic (scales below injection)
- $Re_\ell \to \infty$ (scales above dissipation $\to$ we are in the "*inertial range*")

<u>**Note:**</u> **Komogorov dissipation length scale @ $Re_{\ell d}$=1**

$Re_\ell = U_\ell \ell / \nu$, with $U_\ell = \varepsilon^{1/3} \ell^{1/3}$ $\to$ $\ell_d = \varepsilon^{-1/4} \nu^{3/4}$

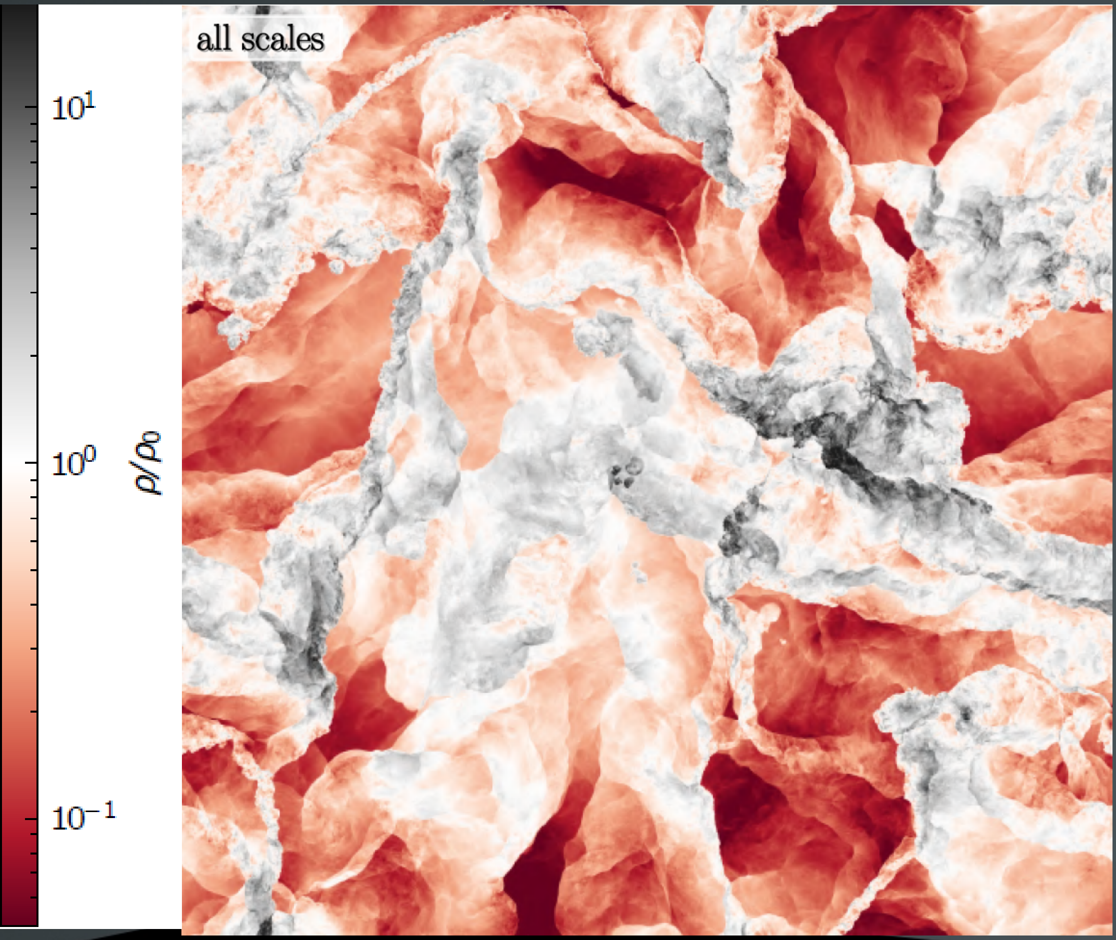
Velocity vs. scale in our Galaxy

Falgarone et al. (2009). $\Pi = \Delta U^3/L = 3.10^{-3} - 3.10^{-4}\ cm^2/s^3$

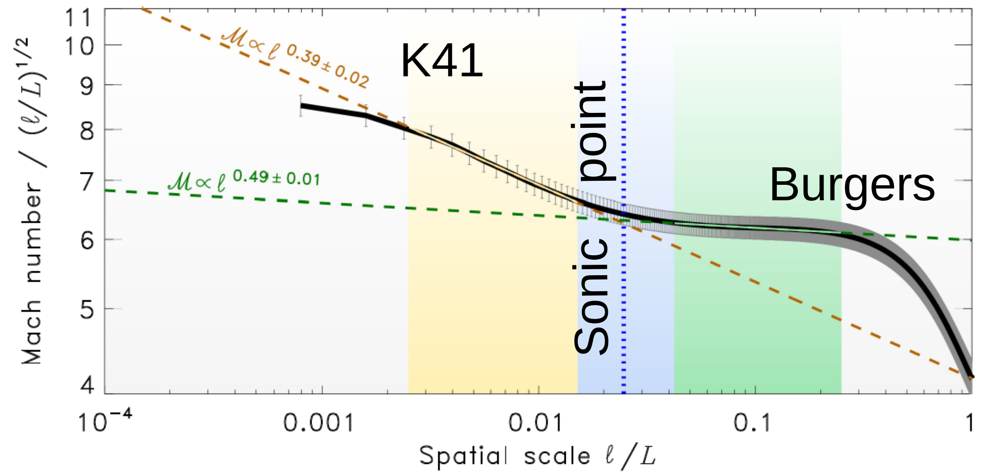


Compressible isothermal turbulence

- Federrath (2021), 10000^3 isothermal simulation !
- $E(\rho^{1/3} u) \sim k^{-5/3}$ (Kritsuk 2007) ?

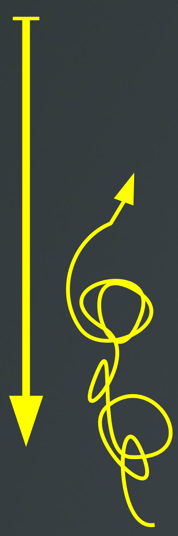


Kolmogorov scaling @ small Mach,
Burgers scaling @ large Mach

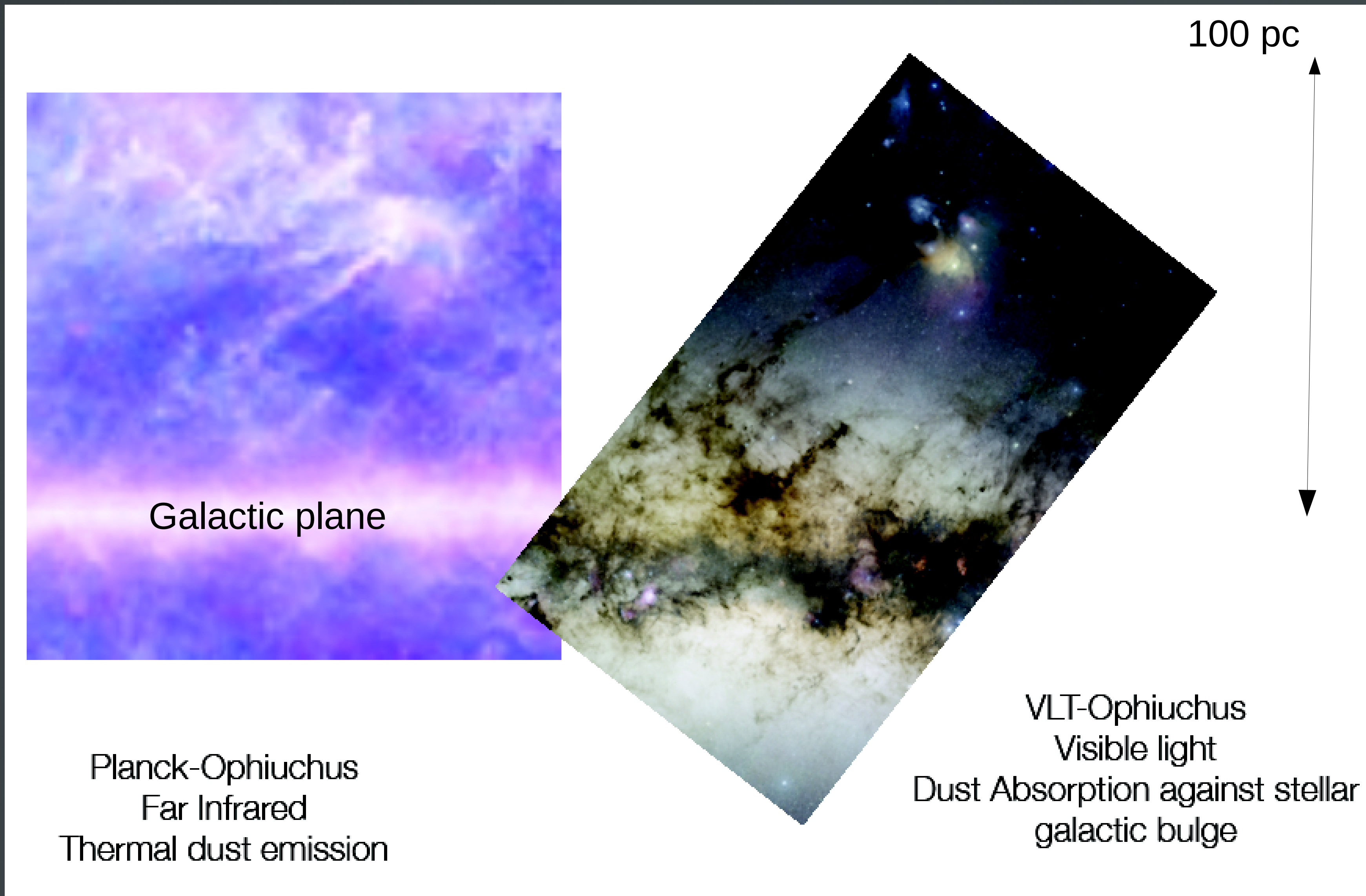


The sky near the galactic center

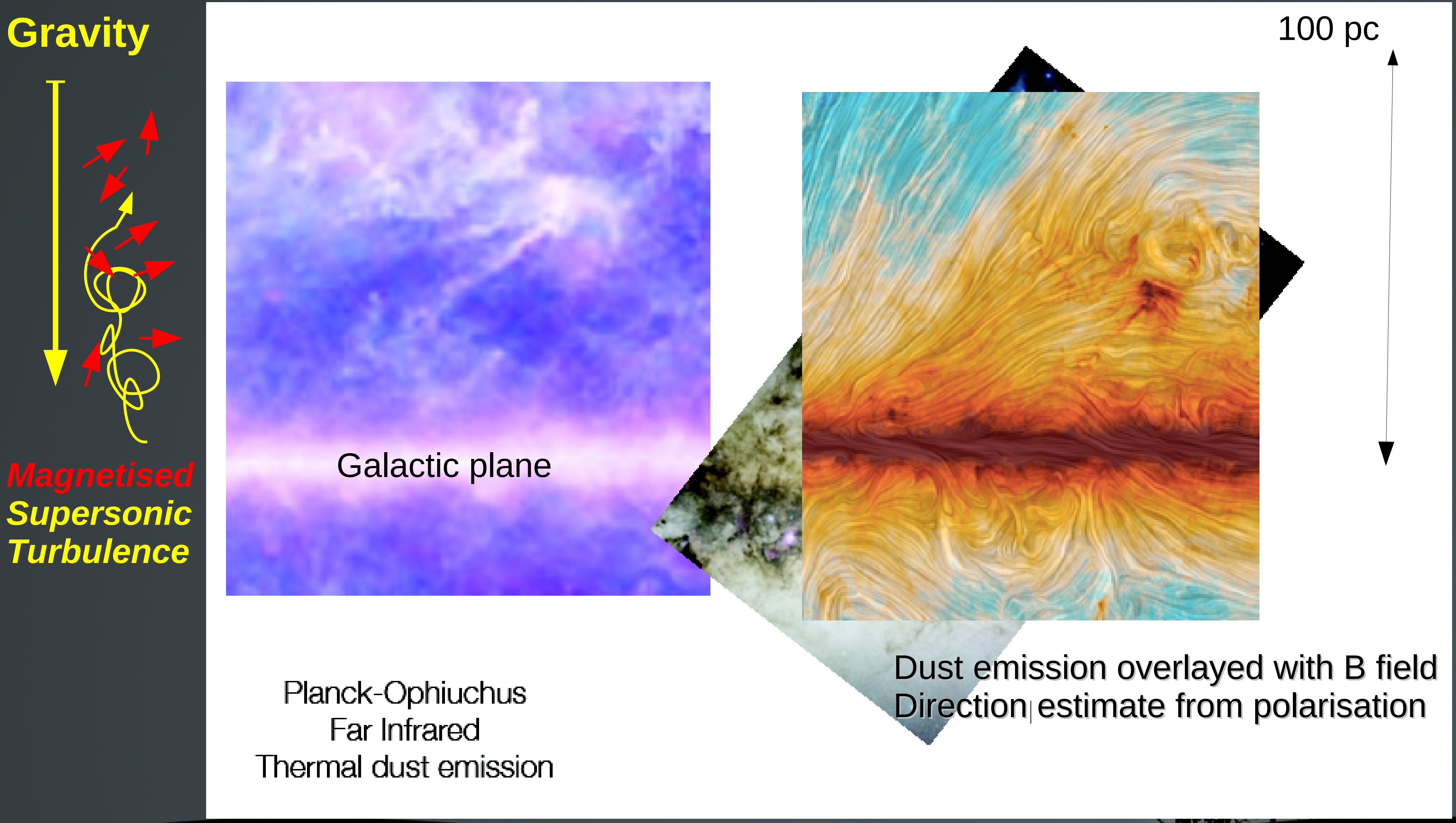
Gravity



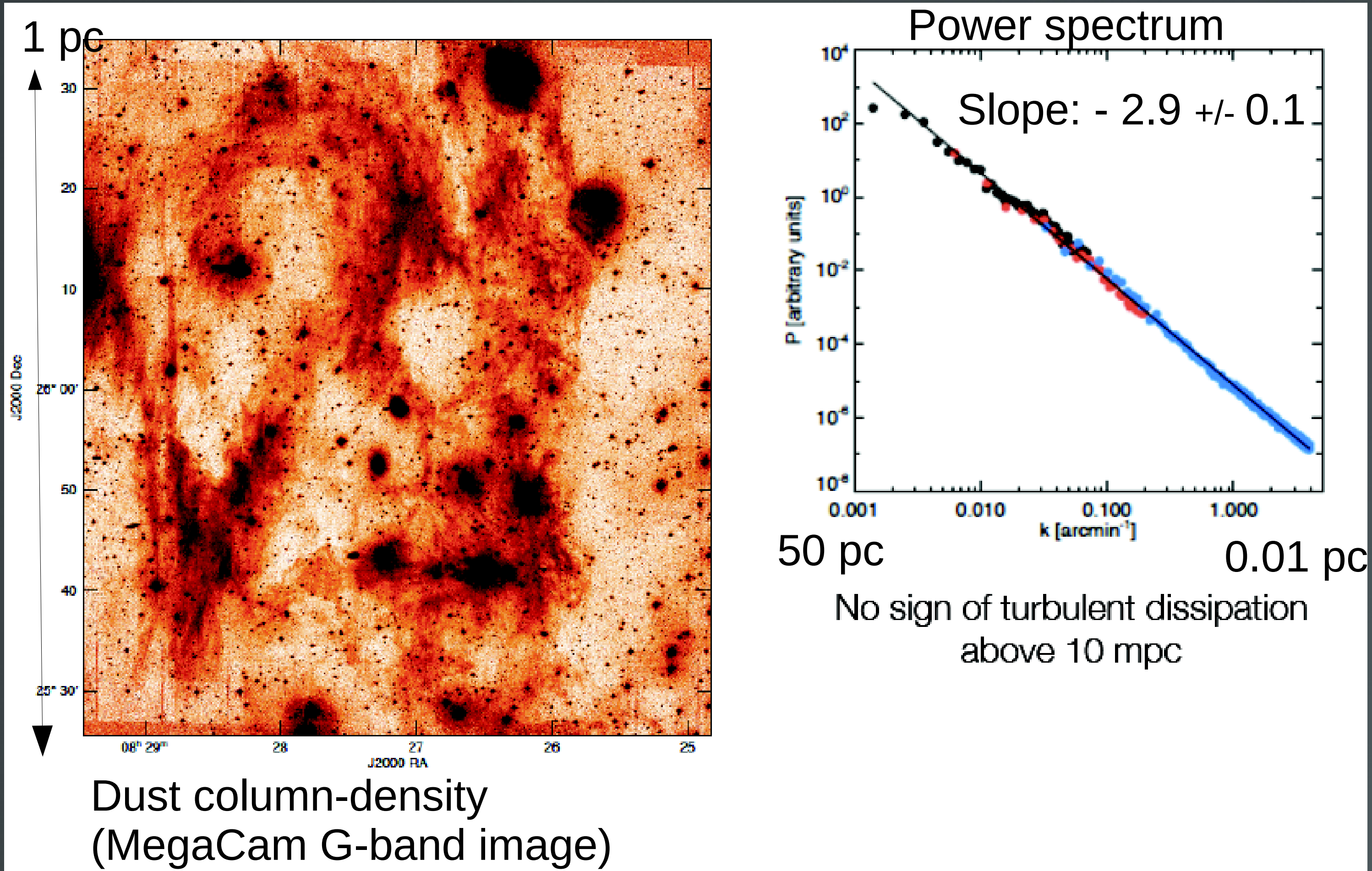
Supersonic Turbulence



The sky near the galactic center



The dust emission spectrum



Miville-Deschênes et al 2016

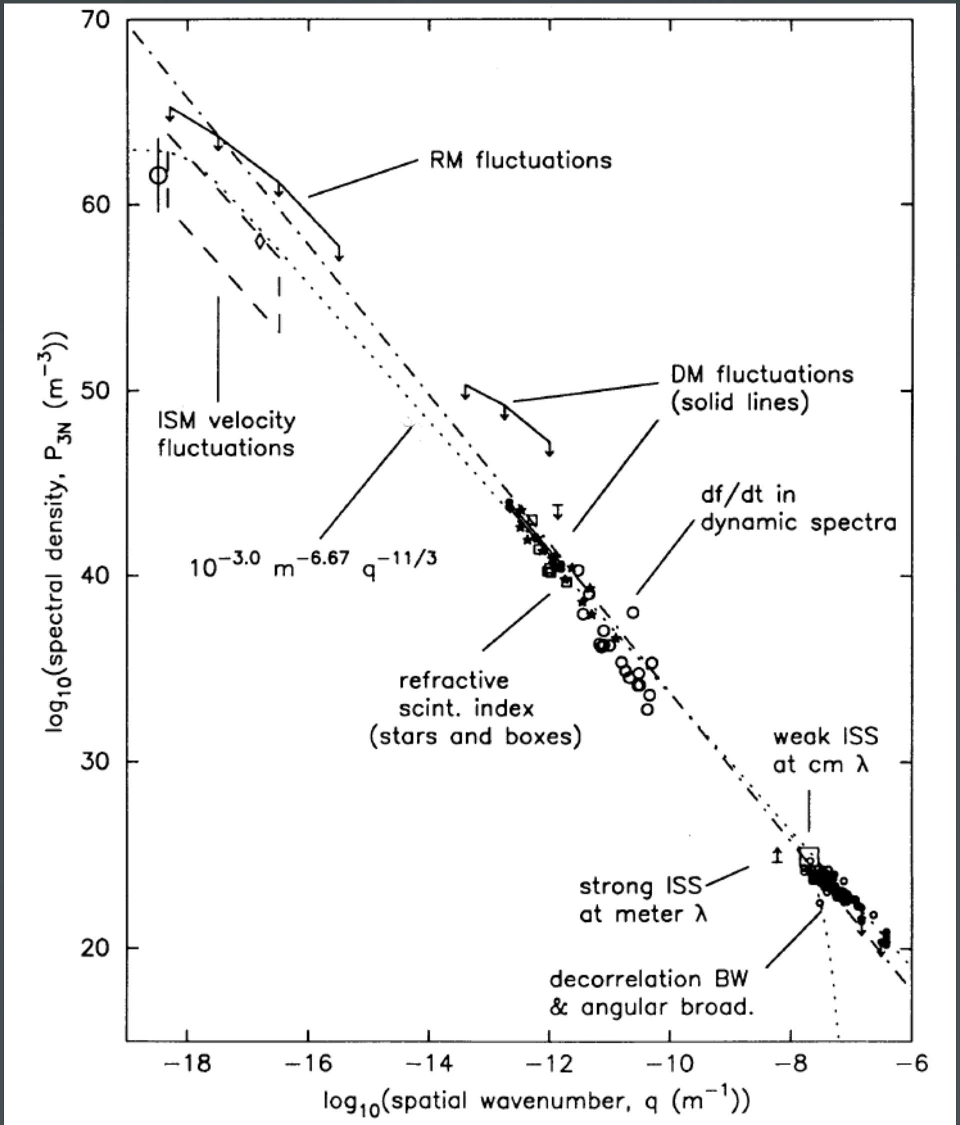
Armstrong (1995)
”The big power law in the sky”

Velocity fluctuations

in the ionised gas

$E_v(k).dk = 4\pi k^2.dk.\mathrm{Fourier}_v(k)$

-5/3 → -11/3



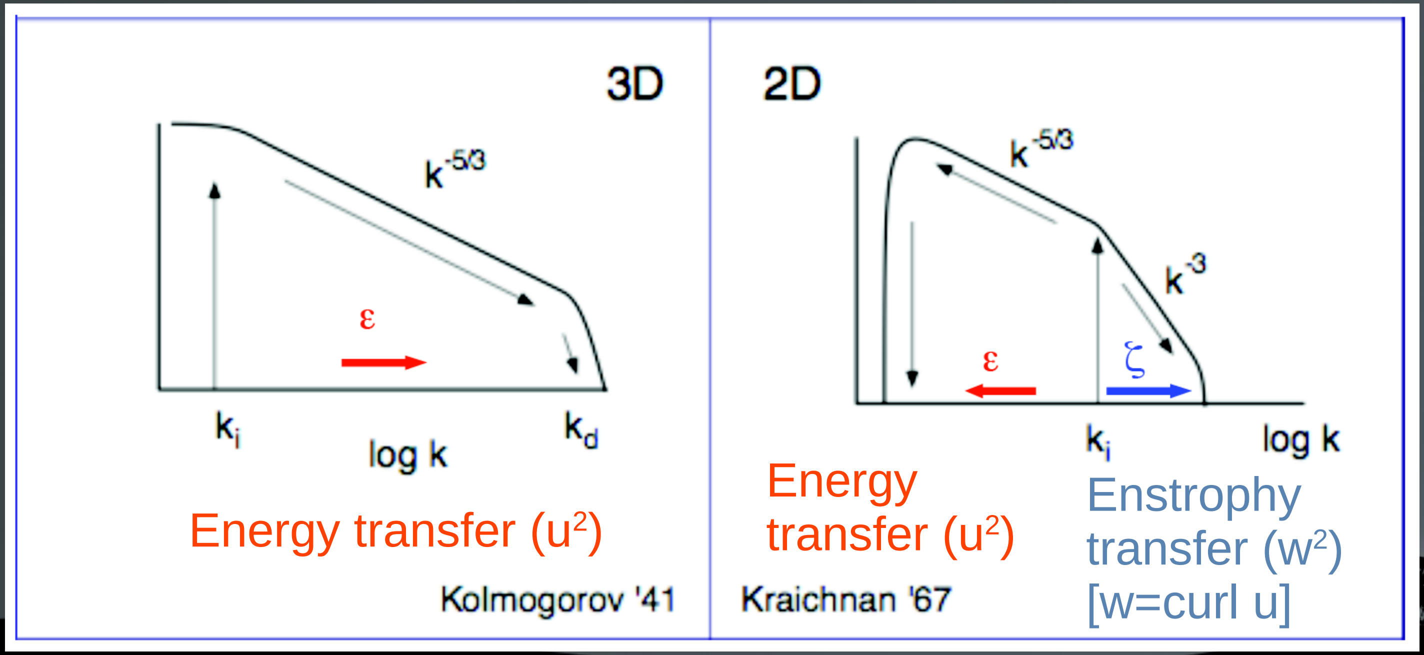
MHD turbulence

- "Reduced" MHD, incompressible MHD, compressible MHD, isothermal MHD, ideal vs. non-ideal MHD (ex: Ambipolar Diffusion) …
- Dynamos: no mean B → mean B
- Iroshnikov (1963) Kraichnan (1965) $E_u(k) \sim k^{-3/2}$
- Goldreich & Sridhar (1995) $E_u(k_\perp) \sim k_\perp^{-5/3}$

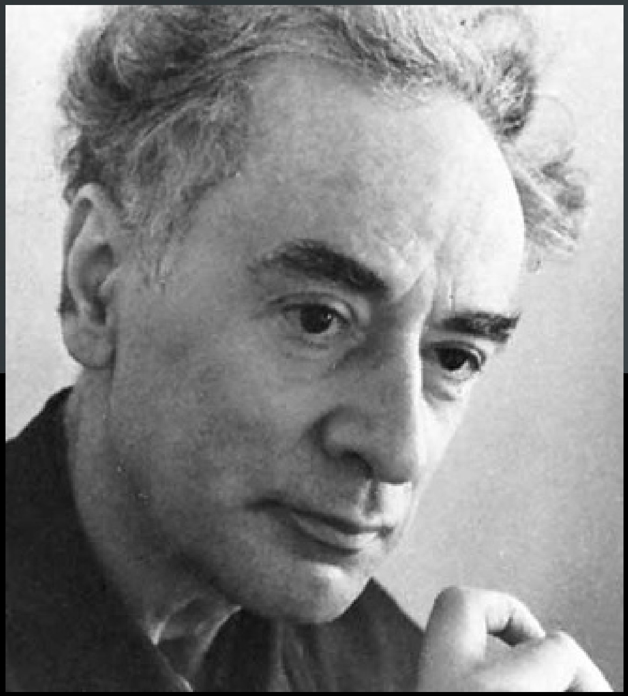
  → Schekochihin (2022) "A biased review" (200 pages ...) reconciles both scalings
- Compressible MHD turbulence ?

2D turbulence

- 2D turbulence: Kraichnan (1965)
- $E_u(k) \sim k^{-3}$ below injection scale (enstrophy cascade)
- $E_u(k) \sim k^{-5/3}$ above, energy cascades to *larger* scales
- Application: Discs, galaxies, atmospheres

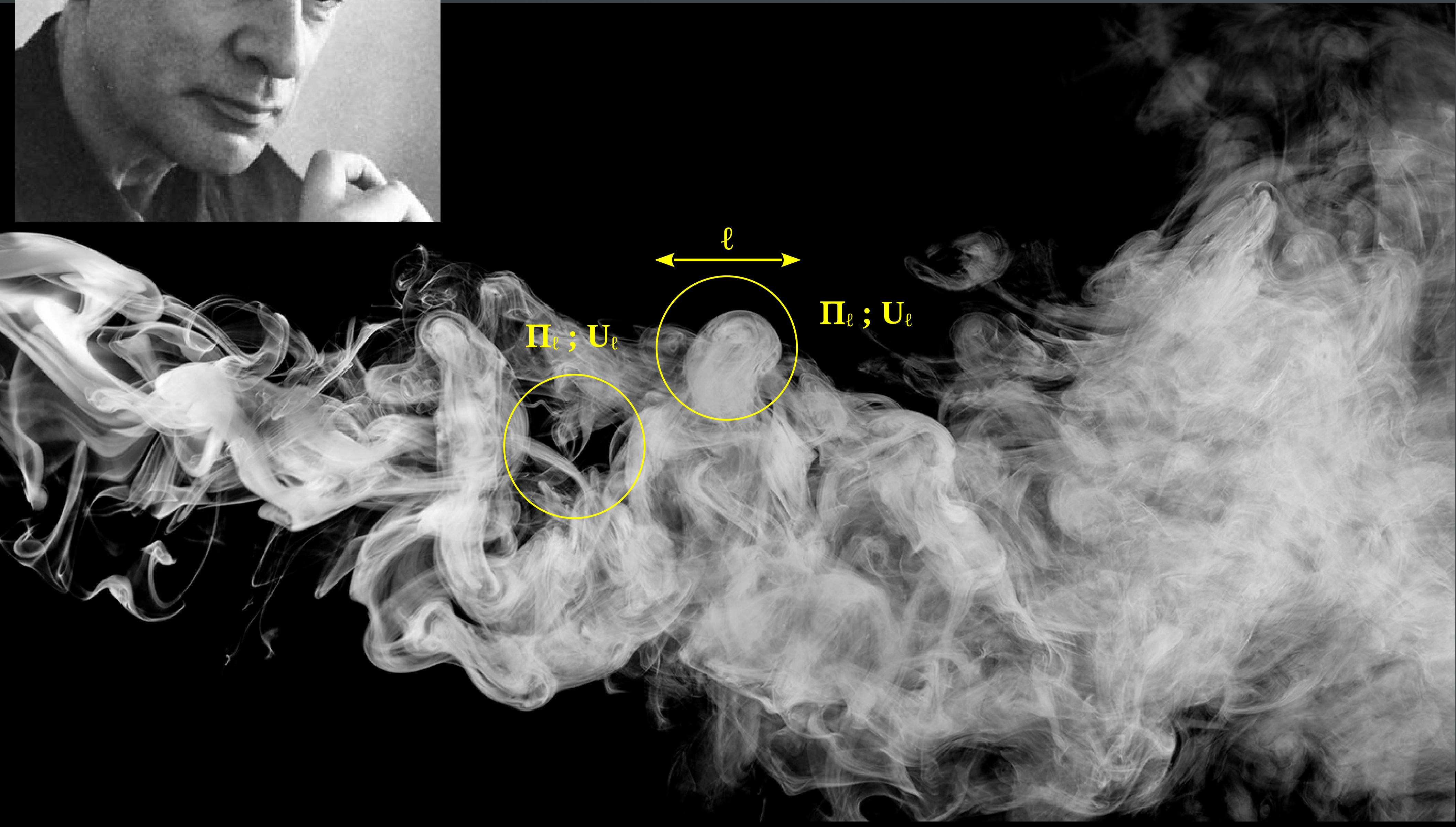


(cf. Lanotte lecture)



Lev Landau's objection to K41

The energy transfer rate should not be homogeneous



Kolmogorov 1962
”Refined similarity hypotheses”
log-normal model

- At given scale ℓ turbulence depends on local Reynolds number $Re_\ell = U_\ell \ell/\nu$
- Assume large Re statistics are universal for rescaled velocity increments $\delta_\ell u/U_\ell$ with $U_\ell=\varepsilon_\ell^{1/3}\ell^{1/3}$
- Assume scaling $<\sigma^2_{\log \varepsilon}>=A+B.\log(\ell/L)$ and

log-normal distribution of dissipation rates ε

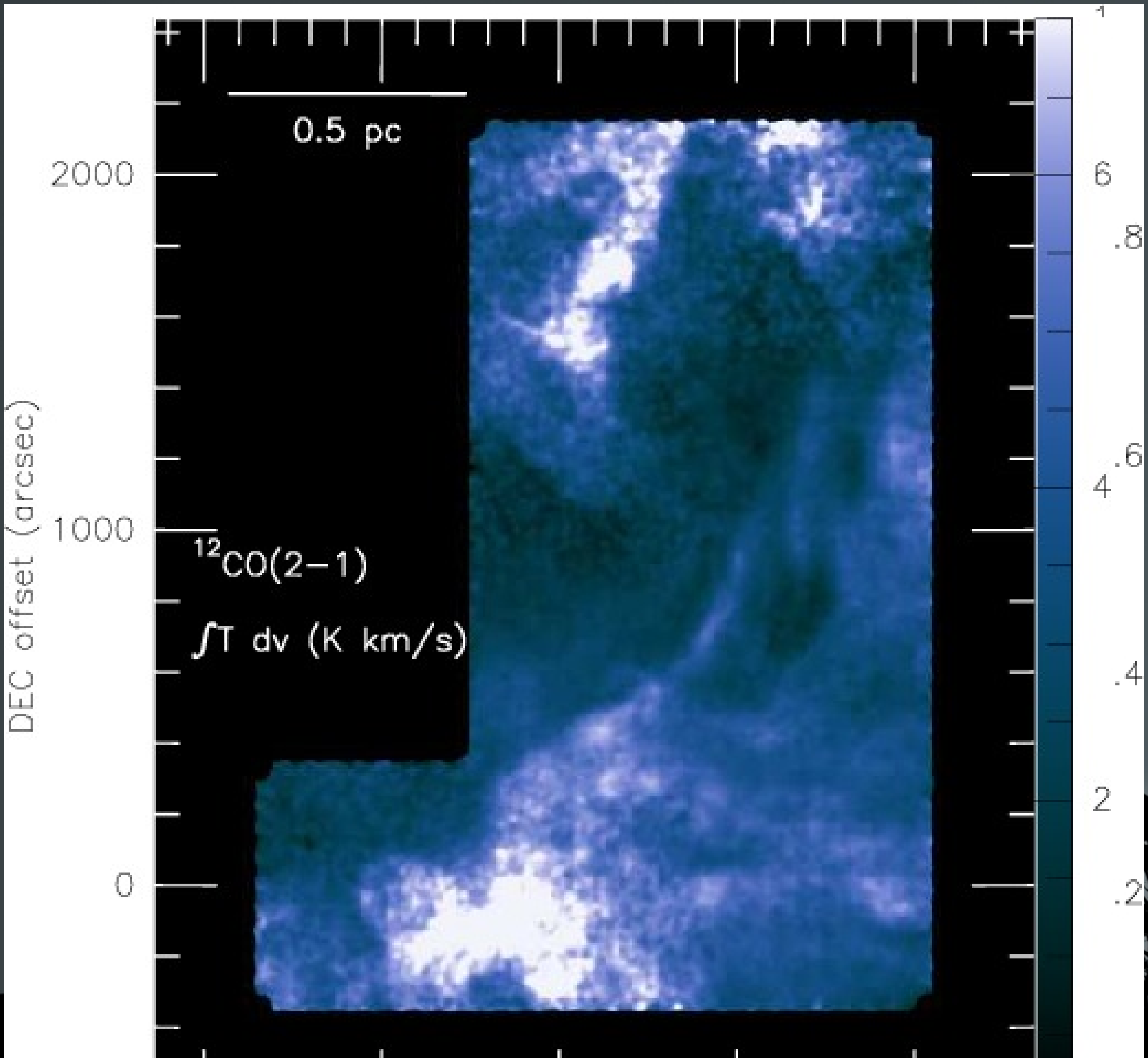
→ $<\varepsilon_\ell^p>\sim(\ell/L)^{Bp\wedge 2}$ and recall $\delta u_\ell=U_\ell.\delta_\ell v[\infty]$

→ $<|\delta u_\ell|^p>\sim(\ell/L)^{p(1/3-Bp/9)}$

Structure functions: $$S(\ell,p) =< |u(x+\ell)-u(x)|^p > \underset{\ell\to 0}{\sim} \ell^{\zeta(p)}$$

Example: Polaris cloud

Hily-Blant et al. (2008)



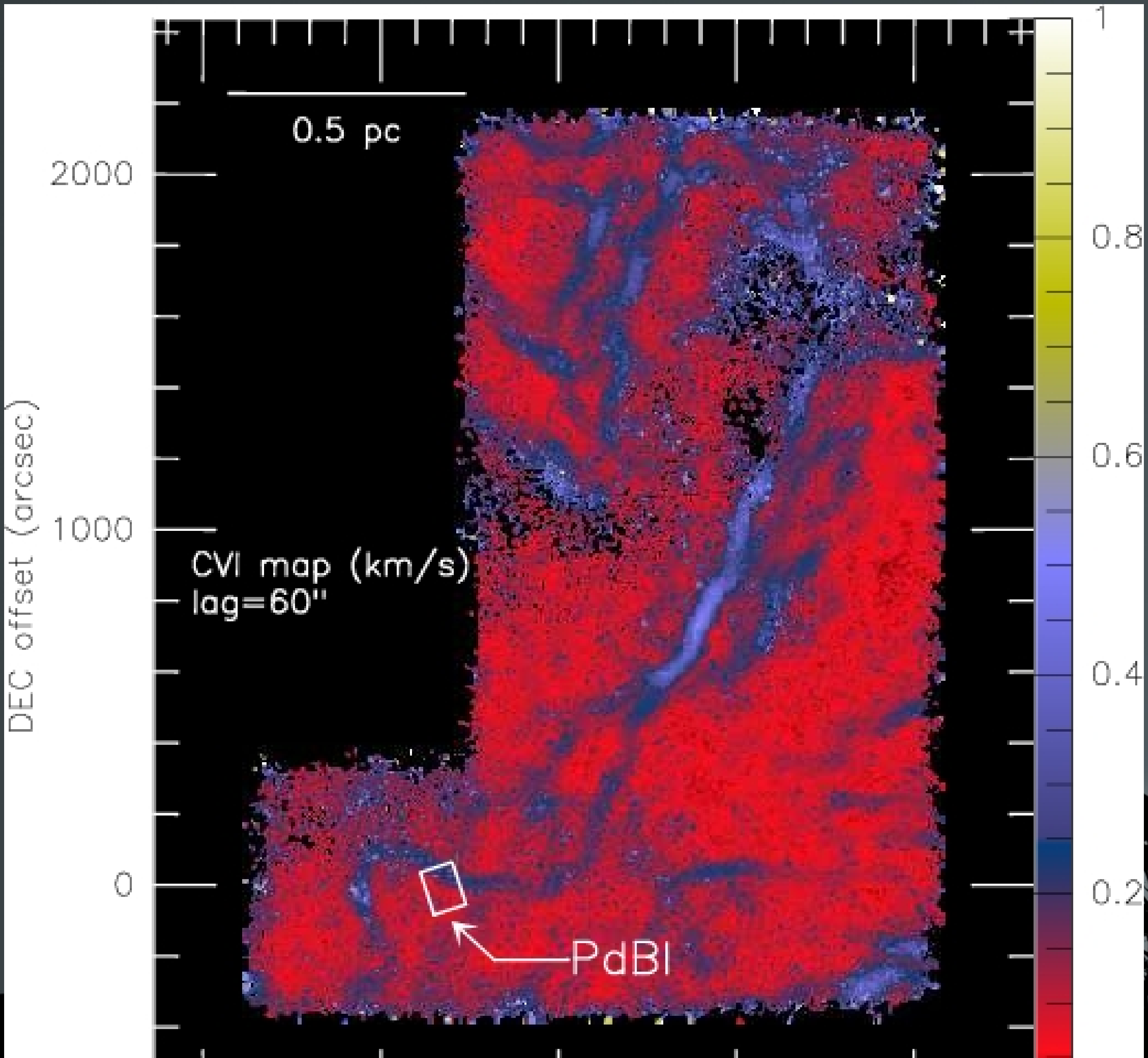
Resolution 12”

Field area:
0.3 square deg

CO by IRAM

Hily-Blant + 2008
Hennebelle &
Falgarone (2012)

Centroid Velocity *Increments*



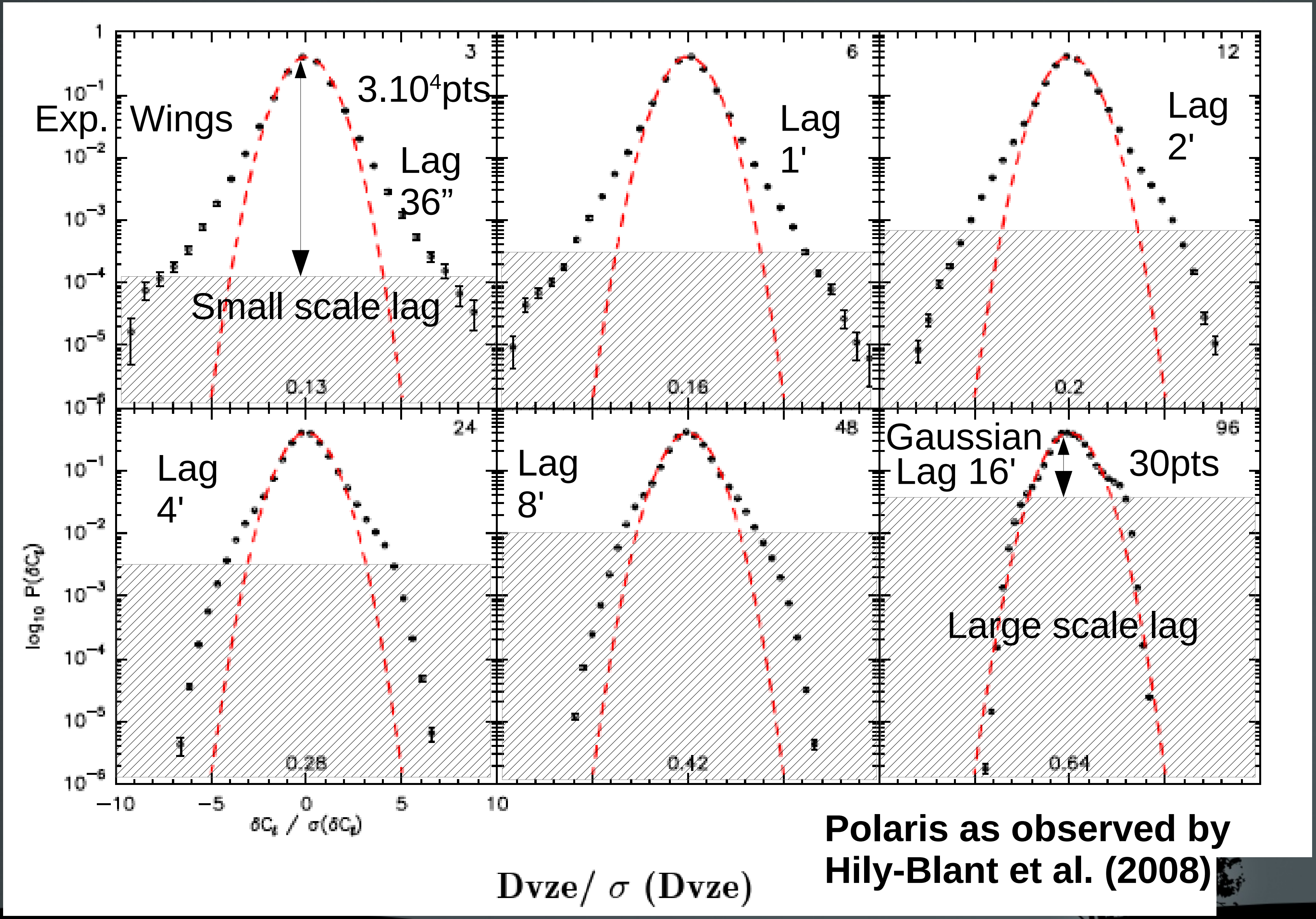
Resolution 12”
Lag 60”

Field area:
0.3 square deg

CO by IRAM

Hily-Blant + 2008
Hennebelle &
Falgarone (2012)

PDFs of Velocity increments in Polaris



Polaris as observed by Hily-Blant et al. (2008)

Intermittency measurement from Hily-Blant (2008) vs simulations

Structure function exponents from observables:

$$S(\ell, p) = < |u(x+\ell) - u(x)|^p > \underset{\ell \to 0}{\sim} \ell^{\zeta(p)}$$

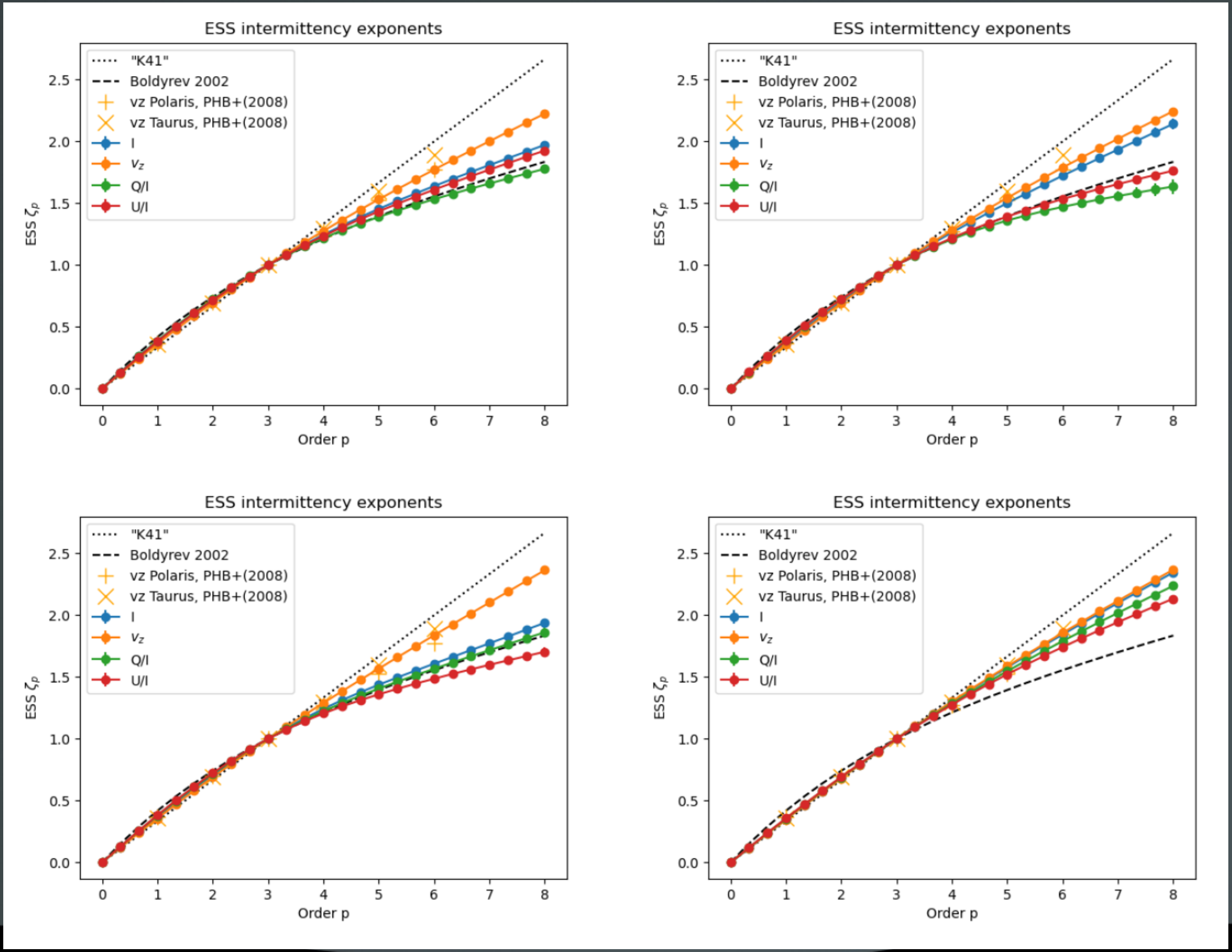
Centroid velocities

Intermittency measurement from Hily-Blant (2008) vs MHD simulations

Orszag-Tang ABC flow

@ early times
(near peak dissipation)

@ “late” times
(at one turnover time)



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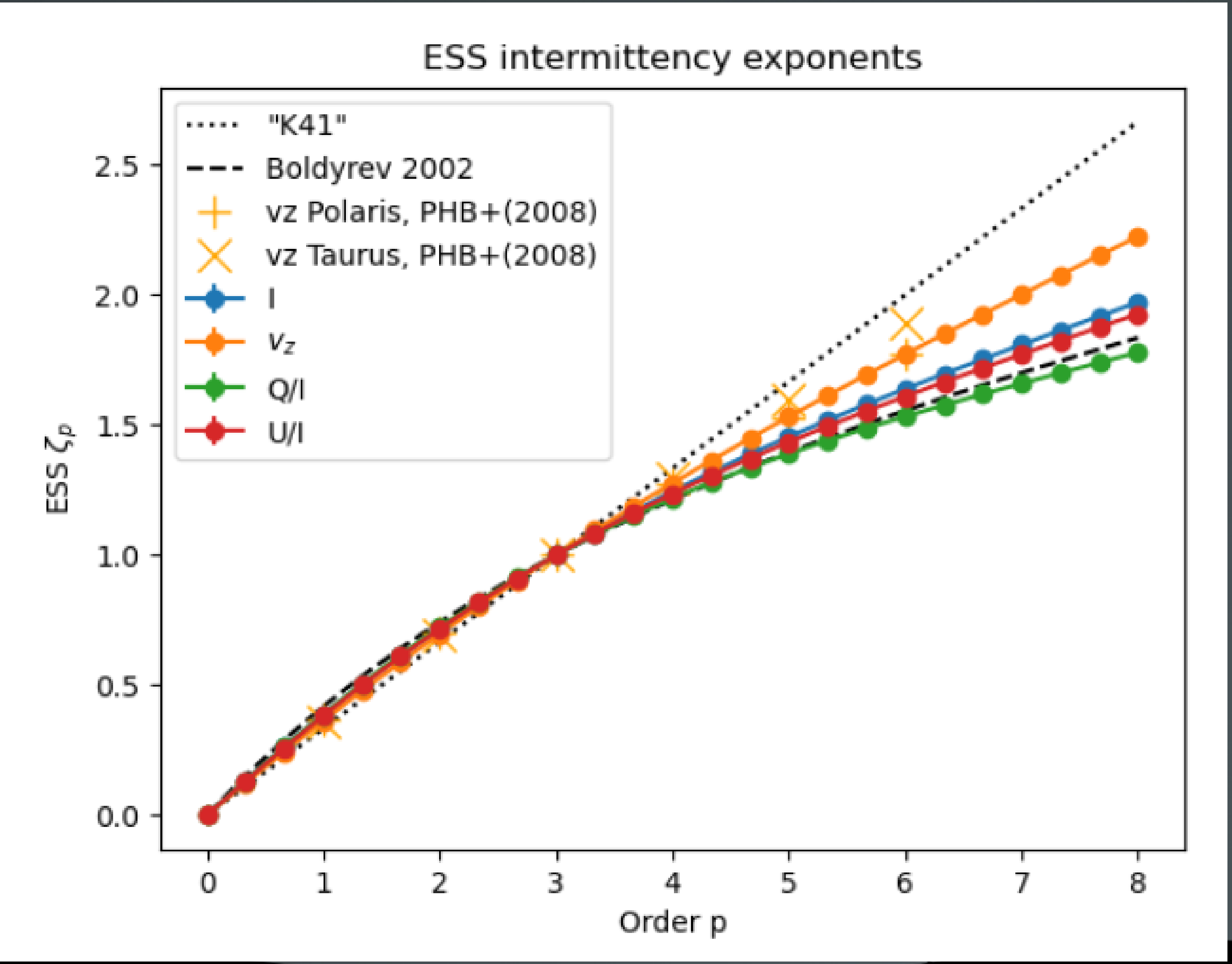
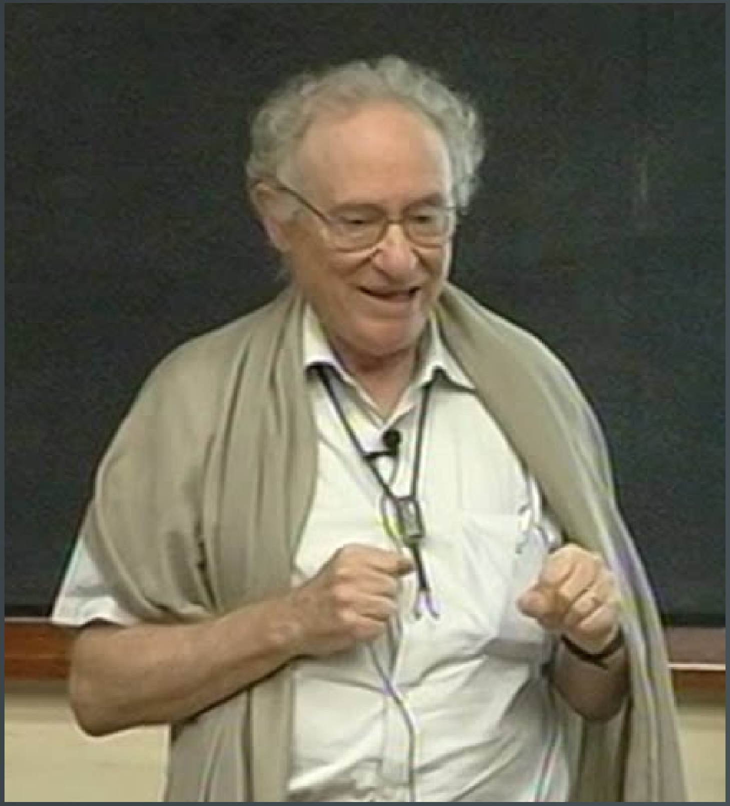
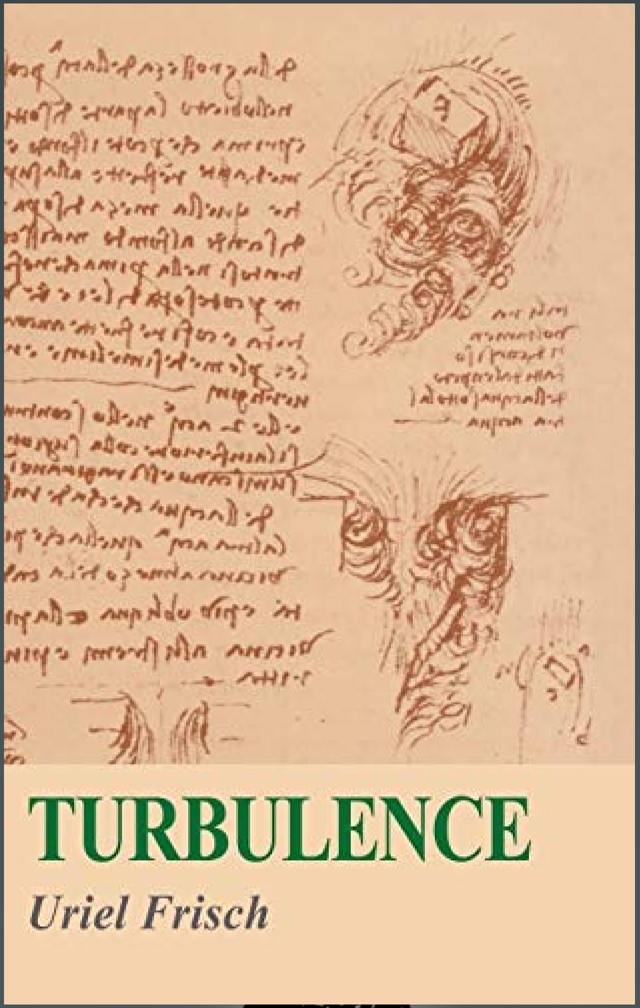


(Lesaffre+2023 in prep.)

Intermittency

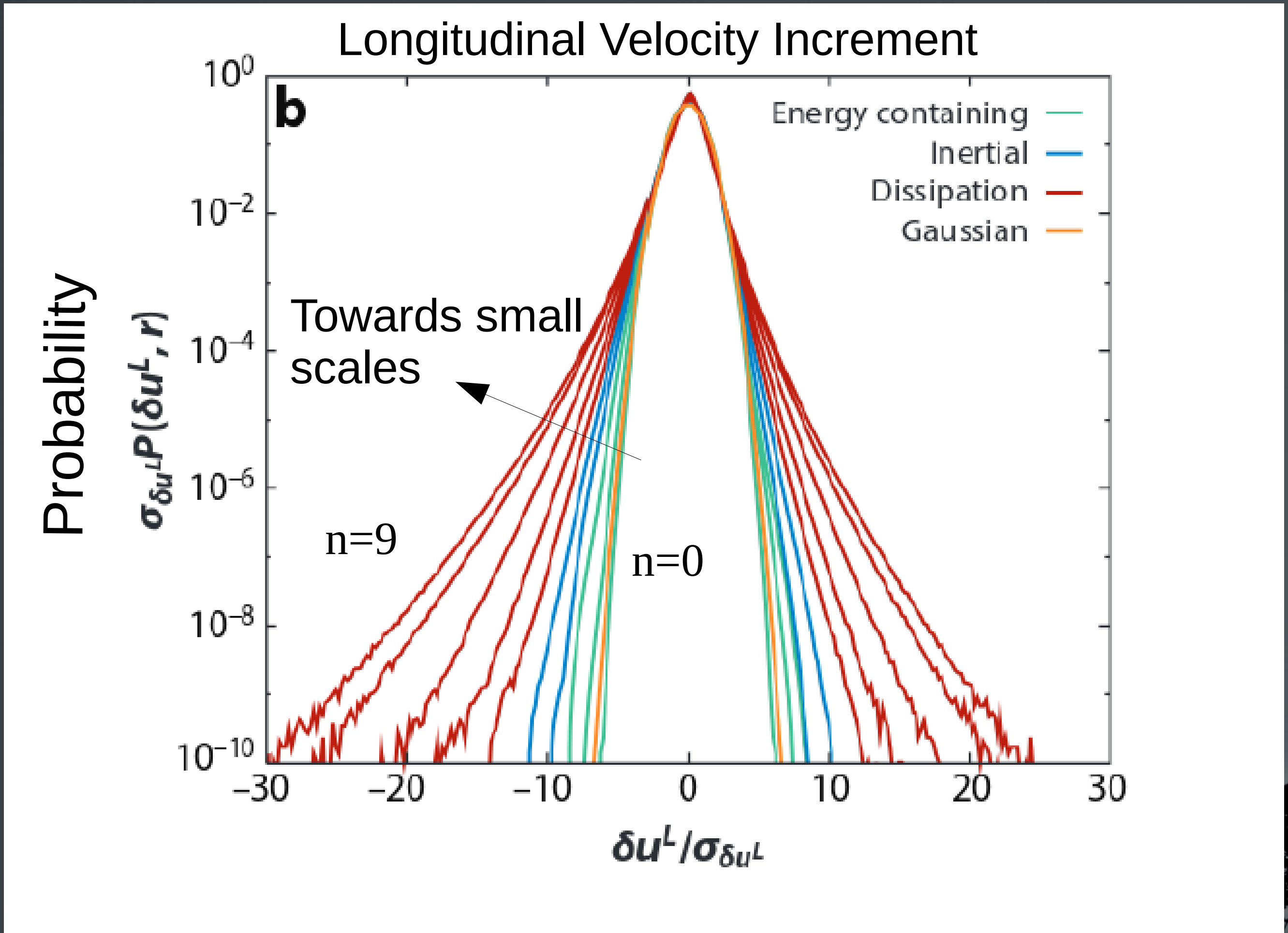
Uriel Frisch





Intermittency: (1) Statistics of increments (PDFs)

- Large deviations are not so rare at small lags



Lag:
$\ell = \ell_0 \, 2^{-n}$

HD turbulence DNS 4096^3 by Ishihara et al. (2009)

Intermittency:
(1) Statistics of increments (PDFs)

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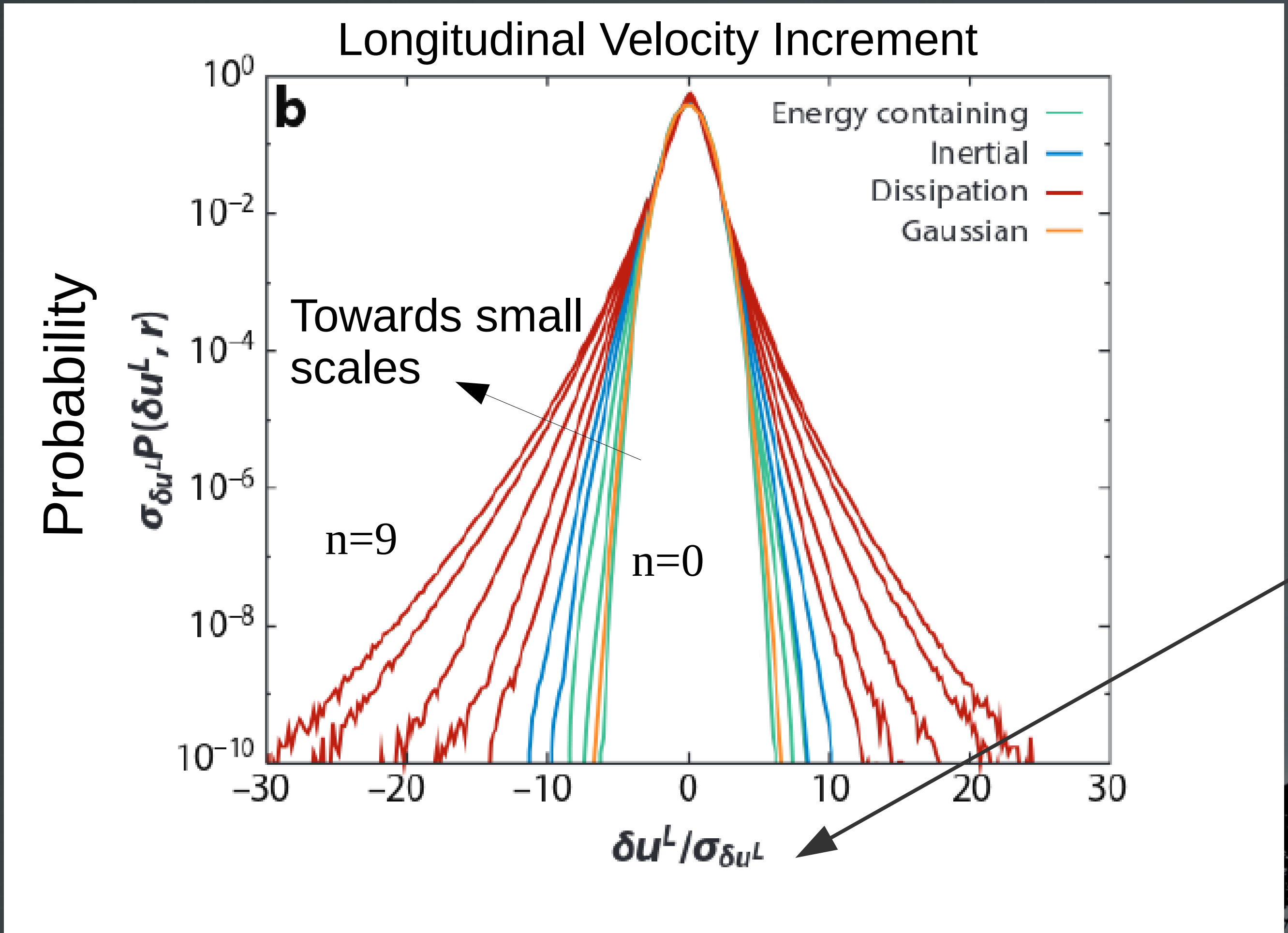


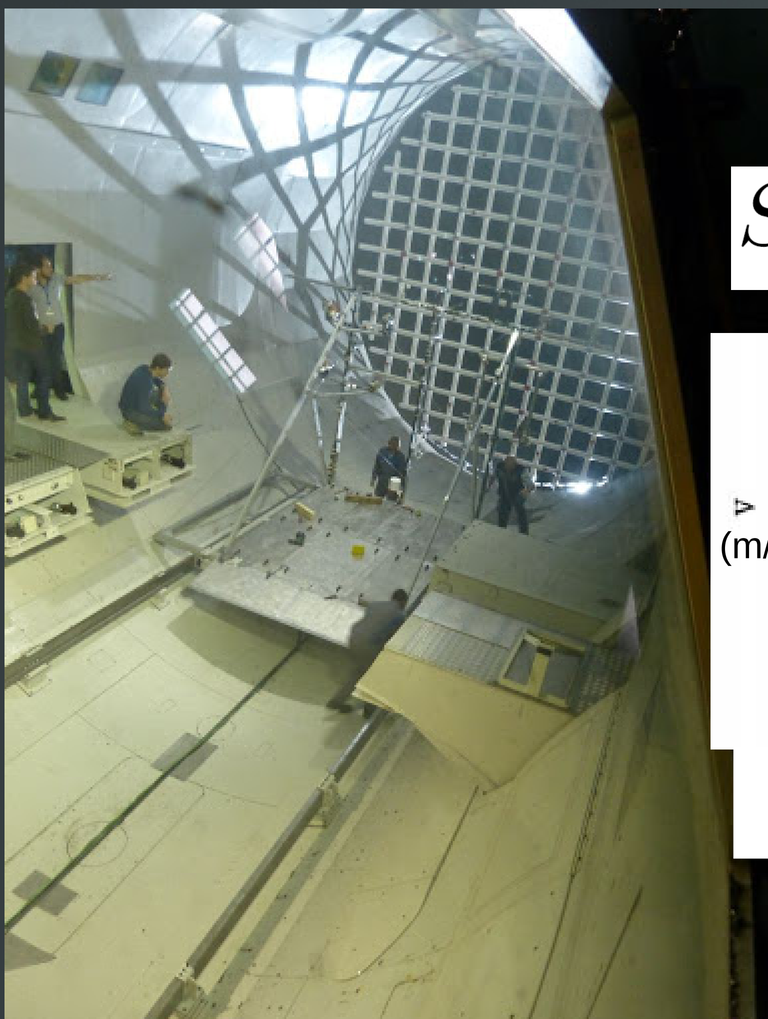
Lag:
$\ell = \ell_0 \, 2^{-n}$

This graph does not show how $\langle \delta_\ell u^2 \rangle$ varies with lag ℓ

HD turbulence DNS 4096^3 by Ishihara et al. (2009)

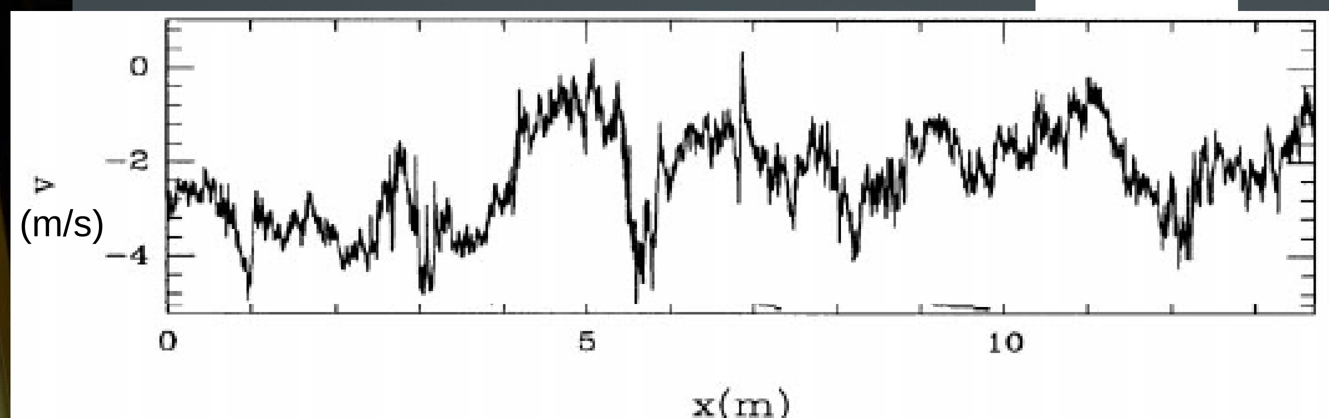
Intermittency:
(2) exponents of Structure functions

- Power-law scalings of increments at small lags



Velocity structure functions exponents: (lag ℓ)

$$S(\ell,p) = \langle |v(x_0+\ell) - v(x_0)|^q \rangle \underset{\ell \to 0}{\sim} \ell^{\zeta(q)}$$

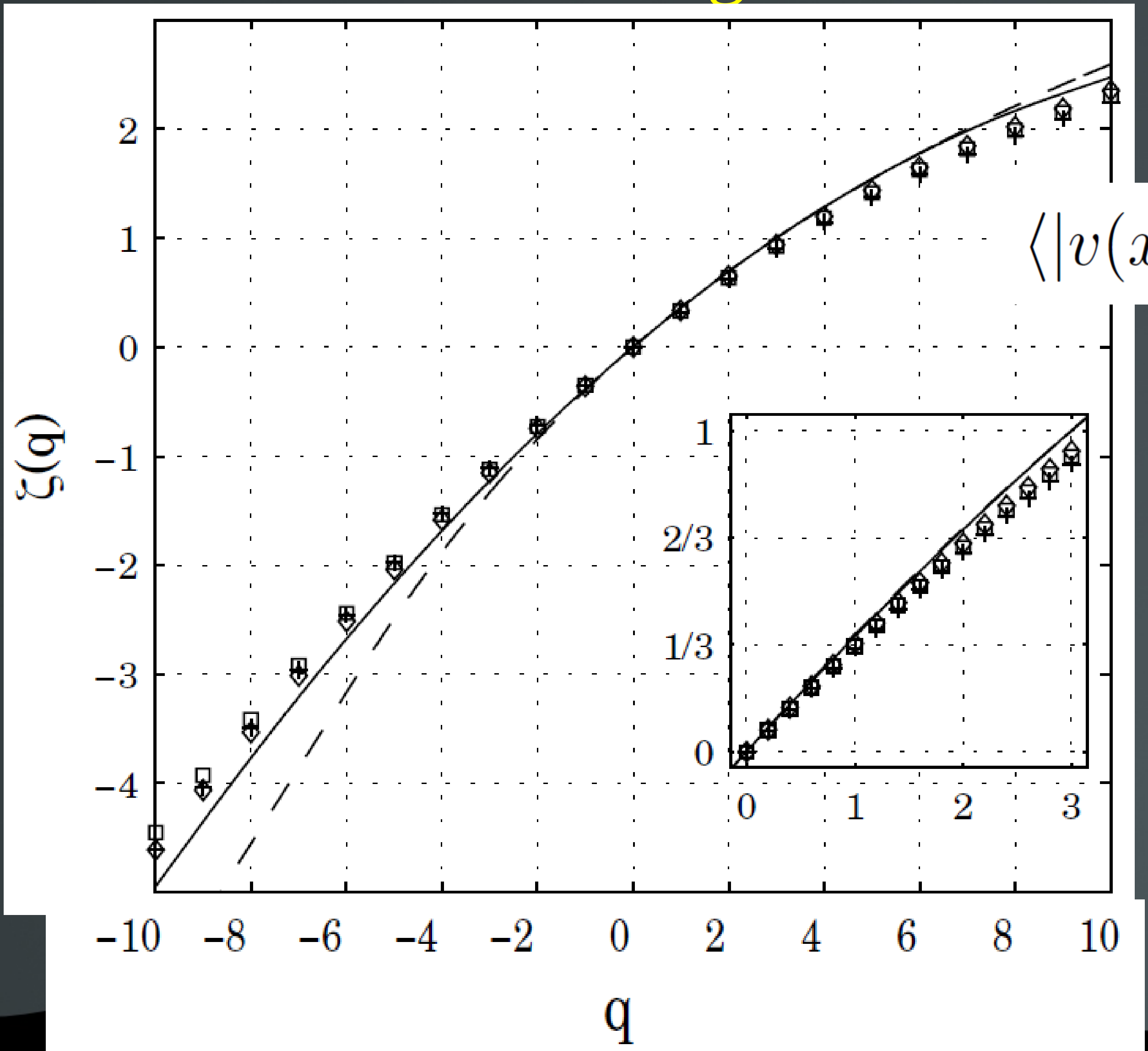


Wind velocity in the Modane tunnel

Lashermes+ (2007)
from Modane wind XP data
(Y. Gagne)

Intermittency:
(2) exponents of Structure functions

- Power-law scalings of increments at small lags



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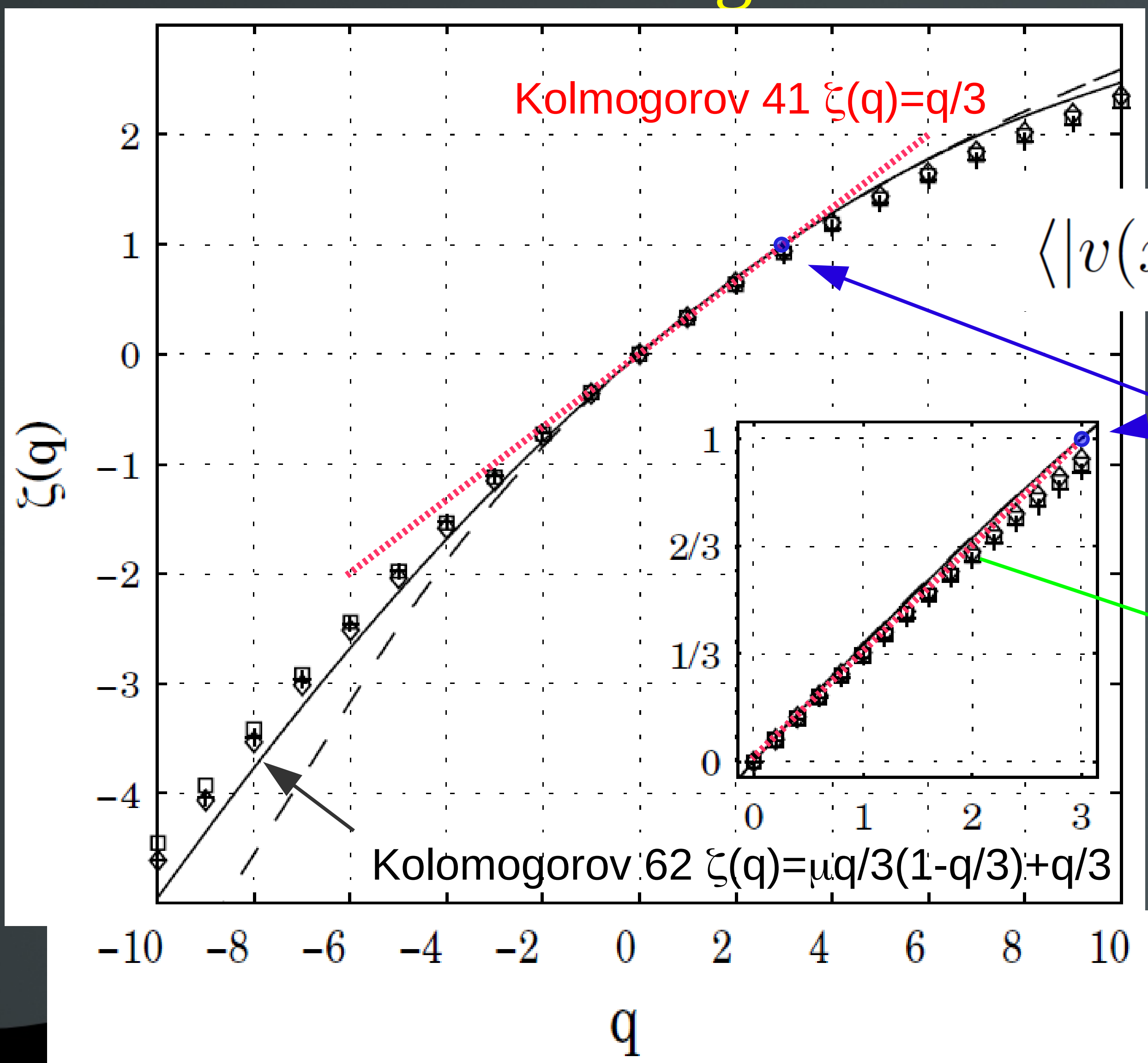


Velocity structure functions exponents:

$$\langle |v(x_0+\ell) - v(x_0)|^q \rangle \underset{\ell \to 0}{\sim} \ell^{\zeta(q)}$$

Karman-Howarth-Monin: $\zeta(3)=1$

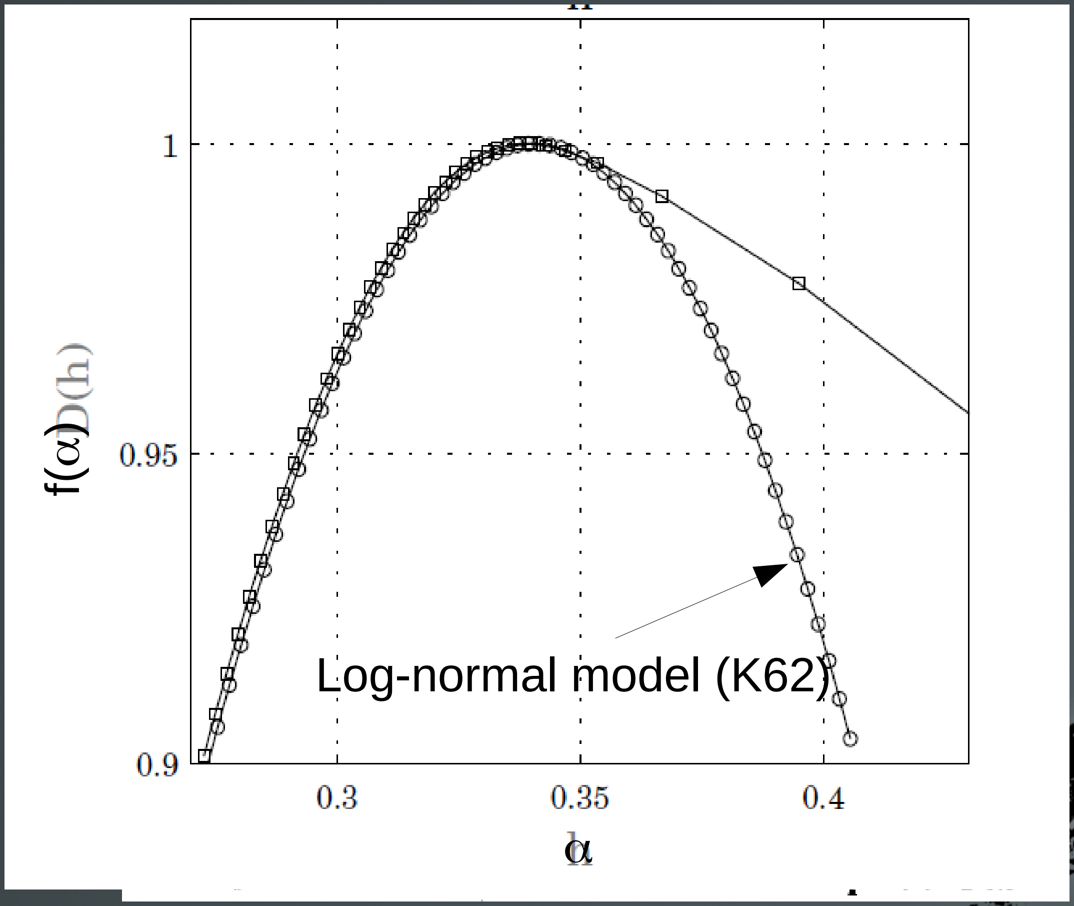
(well... for long[al] signed increments)

Energy spectrum:
$<|\delta_\ell u|^2> \sim \ell^{(2/3+\varepsilon)} \rightarrow E(k) \sim k^{-(5/3+\varepsilon)}$

Lashermes+ (2007)
from Modane wind XP data
(Y. Gagne)

Intermittency:
(3) multifractal spectrum

- Increments "scale as" $|\delta_\ell u| \sim \ell^{\alpha}$ when $\ell \to 0$ on sets with fractal dimension $f(\alpha)$.



Lashermes + (2007) analyse data from Modane wind tunnel

Intermittency:
Various aspects are equivalent

(1) Inc. PDFs ↔ (2) Structure Functions ↔ (3) Multifractals

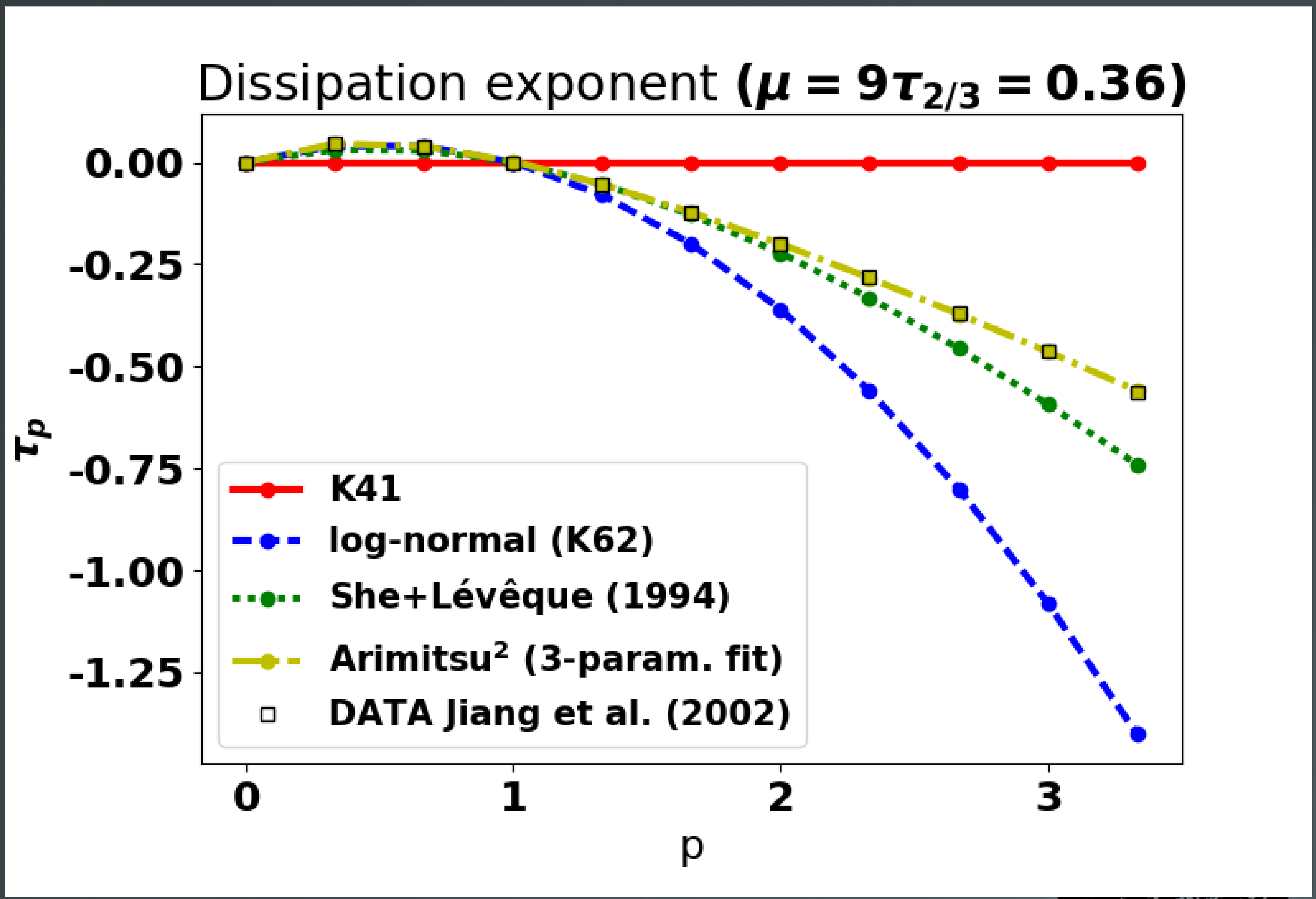
(Large deviation theory, steepest descent argument, moments generating function, …)

- But full equivalence requires full knowledge of the intercepts of the scaling laws: each vision focuses on one aspect of intermittency.
- <u>Note:</u> none of these visions strongly constrains the shape of the dissipation structures. (Generative models as Chevillard+2010 (HD) or Durrive+2020 (MHD) reproduce PDFs but not the coherent structures).

Intermittency: a large variety of models...

- Log-Normal: Kolmogorov (1962), Obukhov (1962)
- She-Lévêque 1994 (generalised Log-Poisson)
- Arimitsu & Arimitsu (2000+)
- Multiplicative cascade and Beta-model (Frish, 1995)
- Hierarchical statistical mechanics (Ruelle 2012)
- Stochastic equations for vorticity (Zybin et al. 2007)
- Multiplicative chaos constructions (Mandelbrot 1962+, Muzy+Bacry 2002, Chevillard 2003+,Durrive+2020)

Intermittency: a variety of models...



$$\langle \epsilon_\ell^p \rangle = \langle \epsilon \rangle^p \left(\frac{\ell}{\ell_0} \right)^{\tau_p}$$

$$\tau_p = \zeta_{3p} - p$$

(Refined similarity hypothesis)

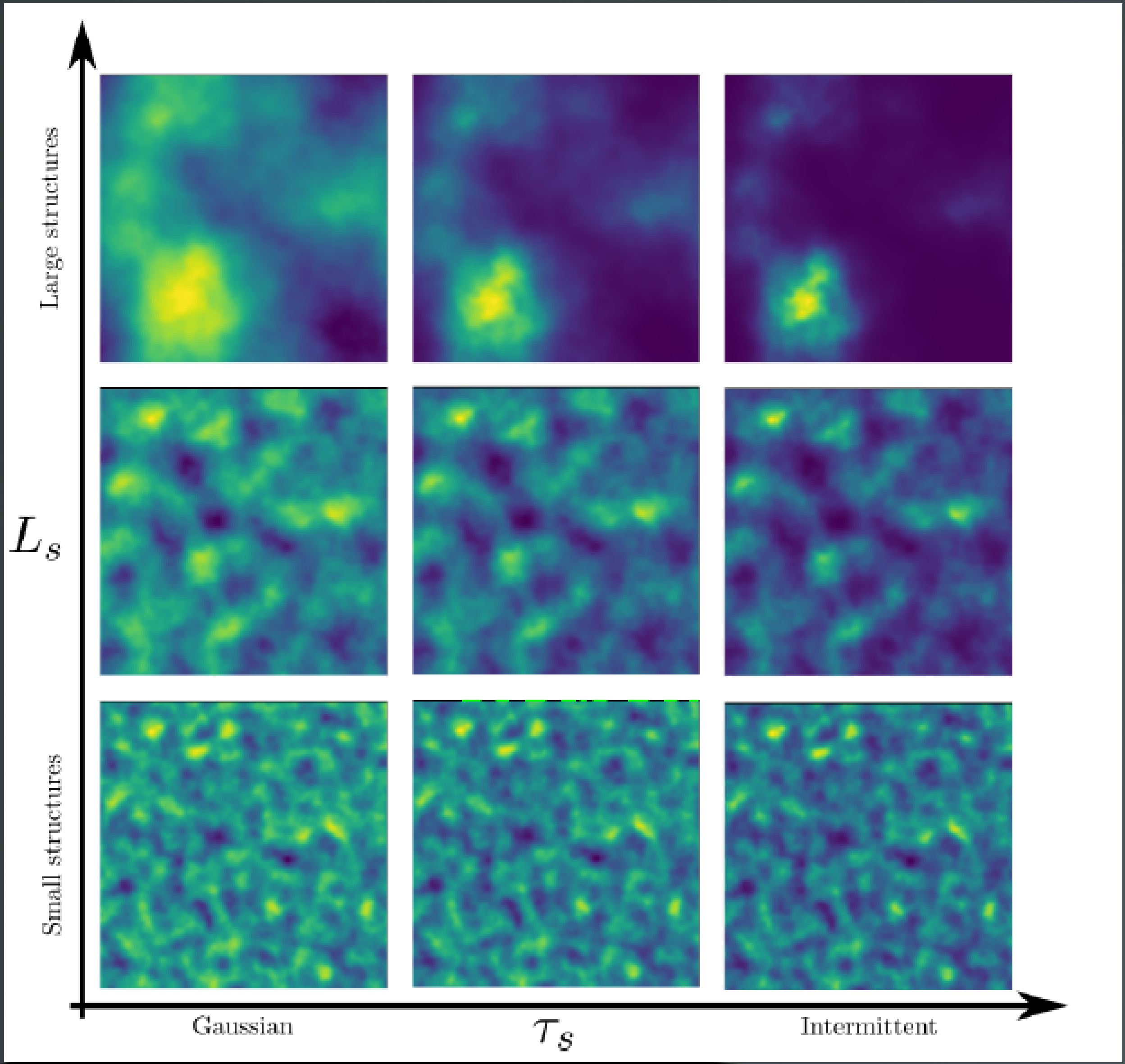
Generative models

- Objective:

  Try to generate random fields which have the known statistical properties of turbulence

“BxC” Durrive, Lesaffre, Ferrière (2020)

Arbitrary spectral index & degree of intermittency, some impact of MHD equations on B,u vectors and their correlations.

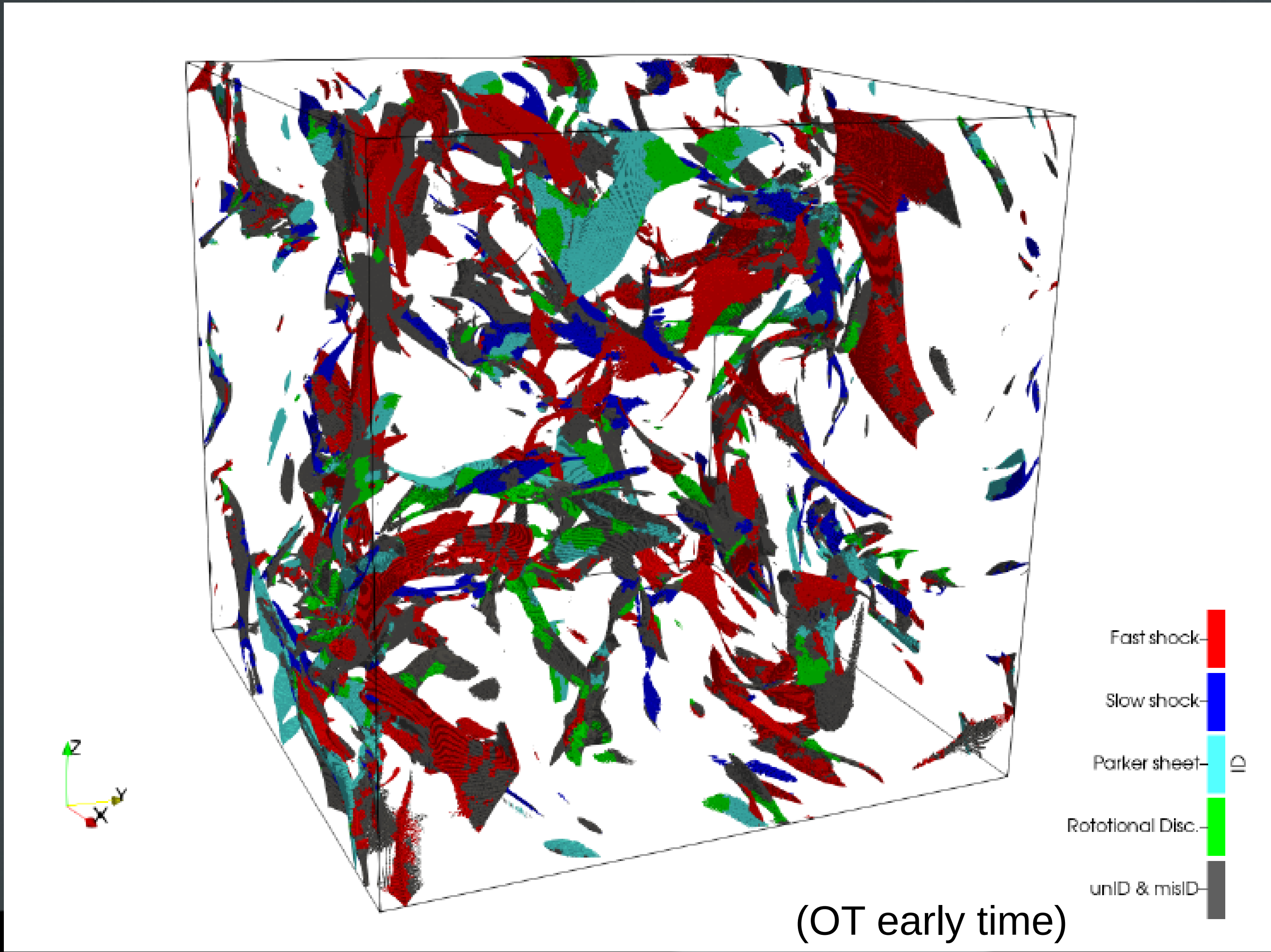


(Note:
2D slices of 3D realisations of a scalar field which all have the same spectra)

The nature of coherent structures in isothermal MHD turbulence

Fast shock Slow shock Rotational Discontinuity Parker sheet

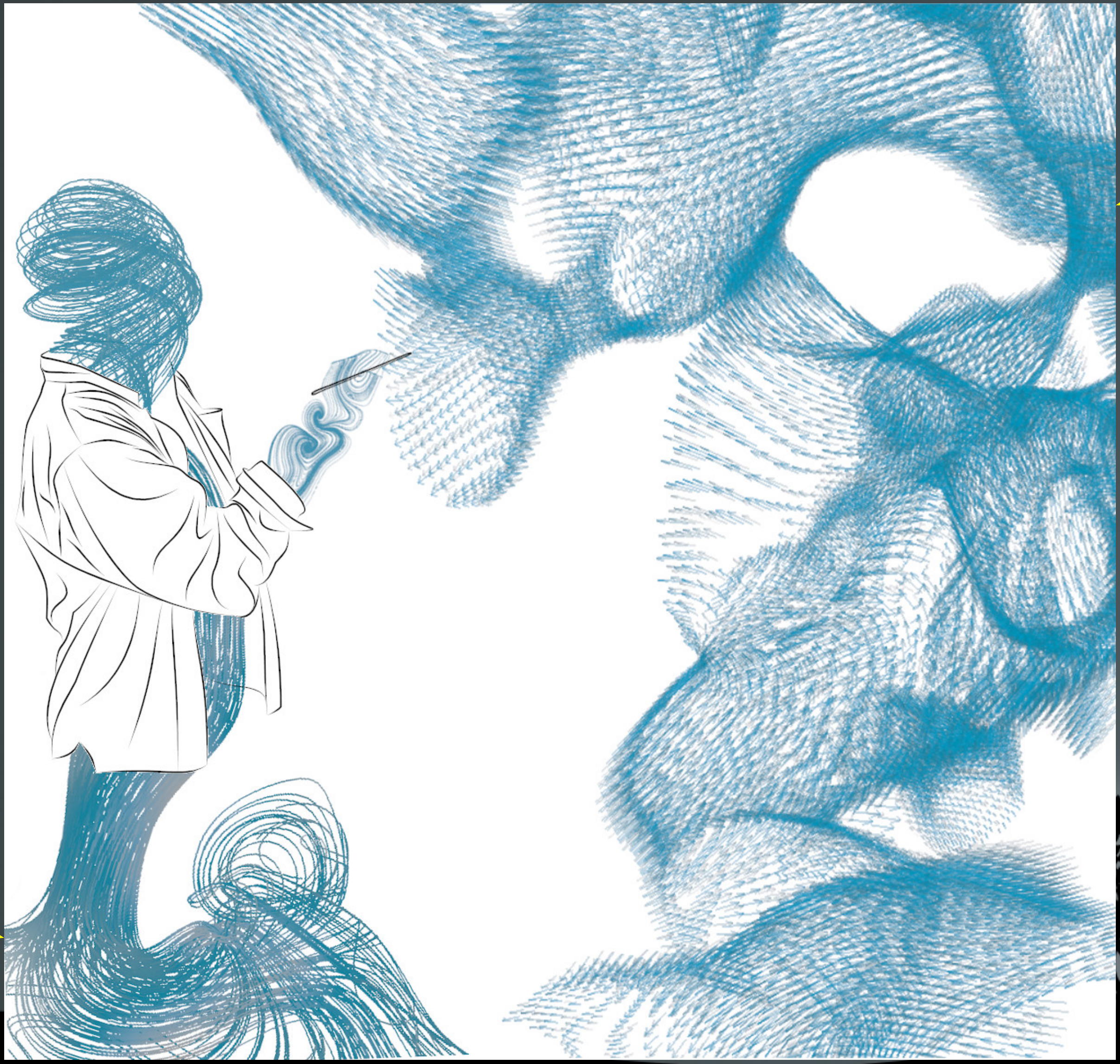
Richard+22



(OT early time)

→ Introduce coherent structures in the B x C model (Durrive+2022)

Pb: structures are specified in a more or less arbitrary fashion (though elegant and well educated ...)



- Magnetic field lines

Velocity field

Image: J-B Durrive

Bibliography
Textbooks on turbulence

- Landau & Lifshitz “fluid mechanics”: on the road to developped turbulence
- Tennekes & Lumley “a first course in turbulence”: phenomenological view
- Frish “Turbulence”: Intermittency
- Monin & Yaglom “ Statistical fluid mechanics”: technical but complete on statistics
- Priest “Magnetic Reconnection”: very pedagogical
- Goedbled & Keppens “MHD of lab. & astro.” : exhaustive on MHD

Howarth-Karman-Monin equation and the 4/5th law

- The only analytical result on turbulence….
- → 4/5th law,energy transfers are towards small scales

$$< (\delta_\ell u_{//})^3 > = - 4/5 <\varepsilon> \ell$$

for *homogeneous isotropic incompressible* turbulence.

- Generalisations of Howarth Karman Monin equation Banerjee & Galtier 2013 (compressible HD and MHD)

- On the HKM derivation: Brachet turbulence lectures **http://www.lps.ens.fr/~brachet/files/Cours_de_Turbulence.html**

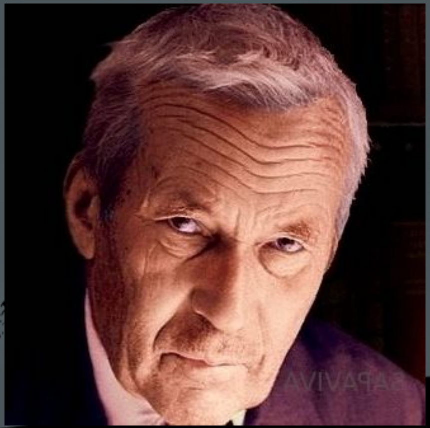
Turbulence summary: mainly Kolmogorov legacy

- Kolmogorov (1941a) : ***power spectrum*** $E_u(k) \sim k^{-5/3}$
- Howarth-Karman-Monin equation → ***energy transfer*** function $< (\delta_\ell u_{//})^3 > = - 4/5 <\varepsilon> \ell$
  for ℓ in the inertial range. Known as the "4/5th law".
- Kolmogorov (1962) : ***intermittency*** $P(\log \varepsilon) \sim$ Gaussian

→ lots of measurements and theories on the statistics of increments $\delta_\ell F = F(x + \ell) - F(x)$

- Importance of coherent structures

Andreï Nikolaïevitch Kolmogorov's Legacy



Shock Waves

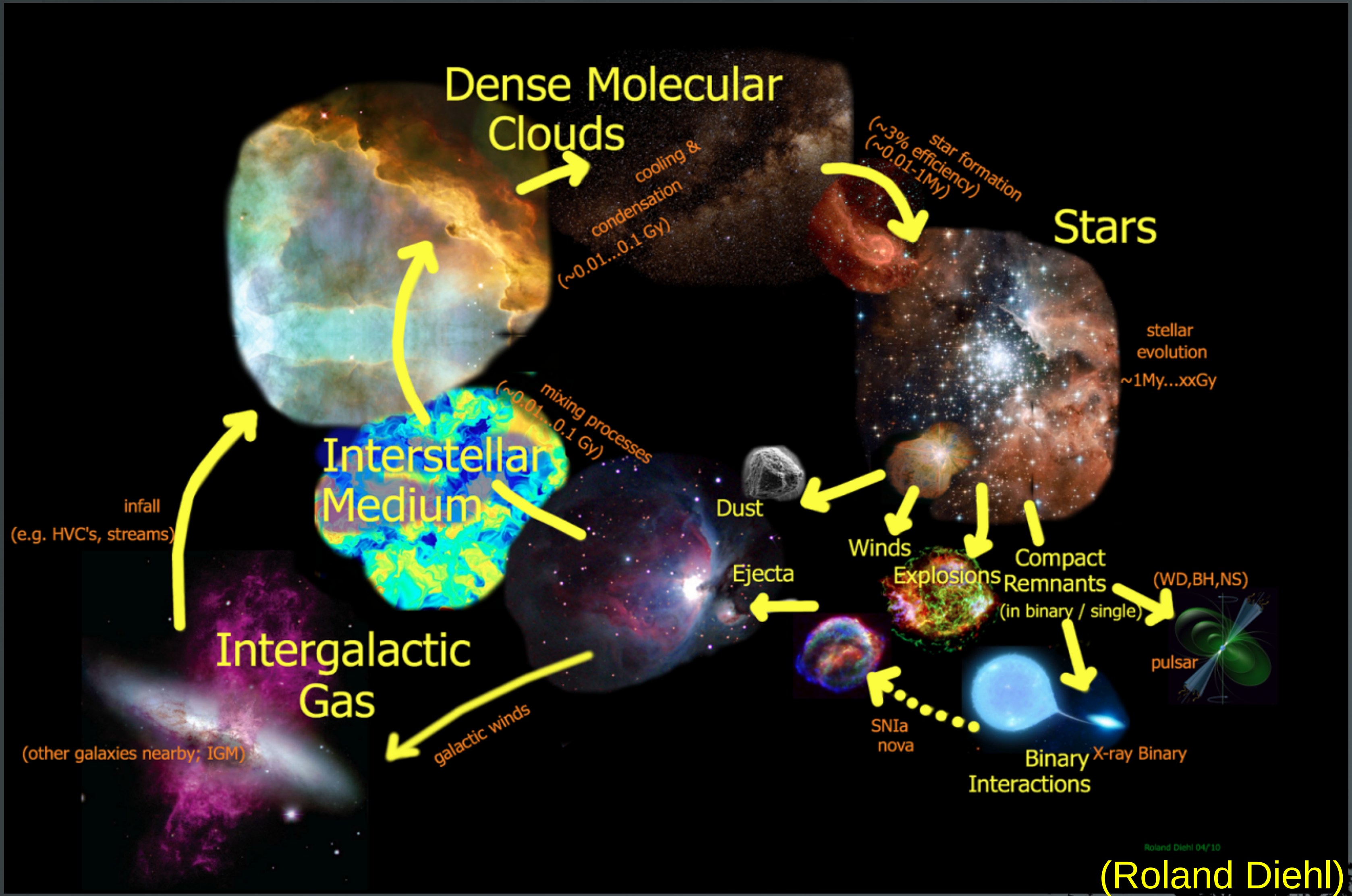
P. Lesaffre
(cf. Les Houches 2022)
Thanks: Antoine, Benjamin, Tram, Thibaud, Andrew

Special thanks: **Jean-Pierre Chièze**

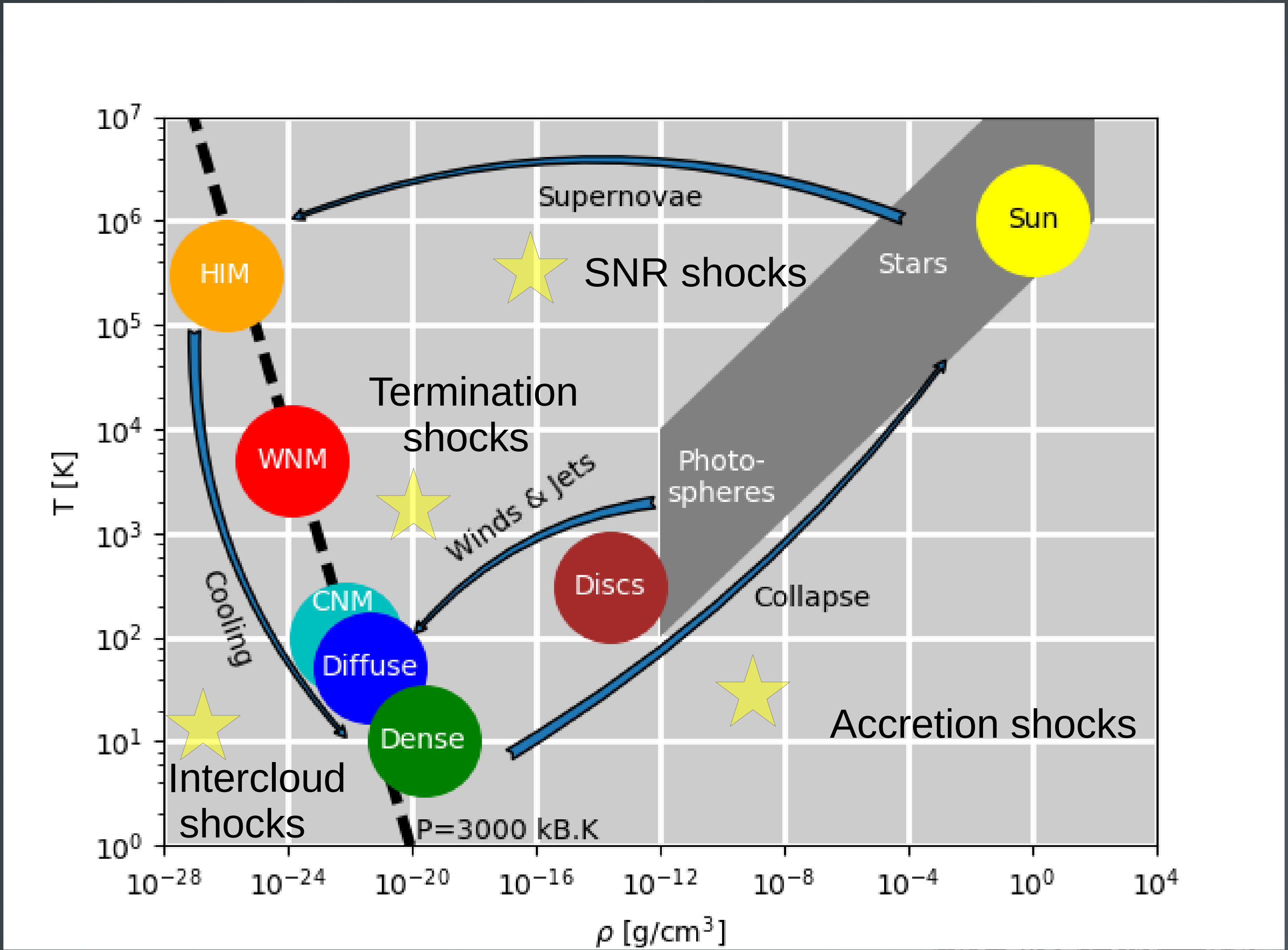
Outline part II: SHOCKS

- Intro, turbulence injection problem
- Brief fluid dynamics reminder
- Waves (linear → steepening → shock waves)
- Shock waves:
  - Jump (Rankine-Hugoniot)
  - Internal structure
  - Shock types
  - Stability
  - Steady shocks and the Paris-Durham shock code
  - Shocks in more than 1D
  - Applications to observations

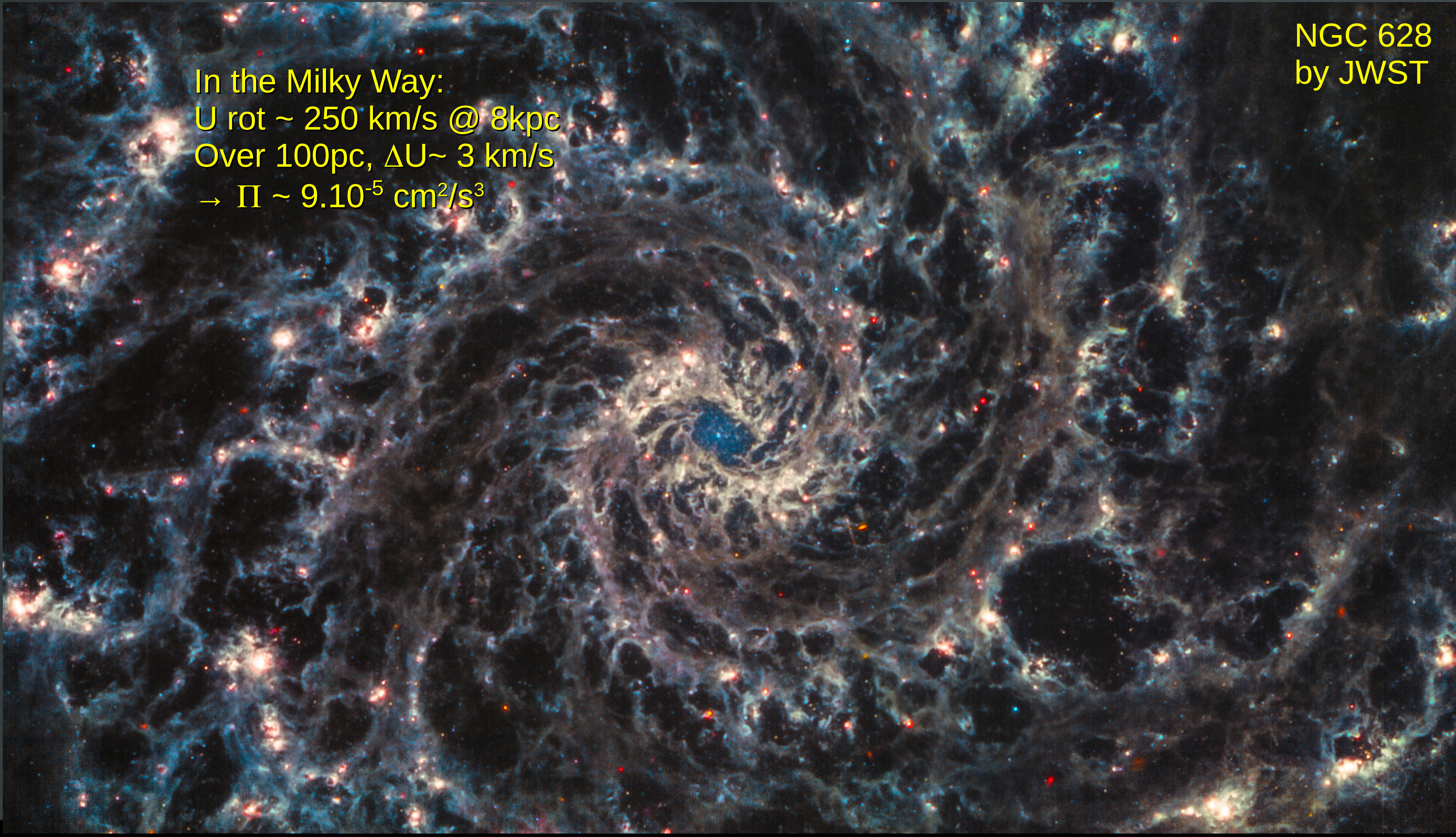
The matter cycle in the galaxy



Quantitative view of the galactic cycle



Turbulence injection by galactic differential rotation



Gravitational energy injection

- Cloud – cloud velocities from virial @ L=100 pc
- $2E_{kin} \sim - E_{pot} \sim \rho GM/L$
- $\Delta U^2 \sim GM/L \rightarrow \Delta U \sim 6.5$ km/s
- $\Pi = \Delta U^3/L \sim 9.10^{-4}$ cm^2/s^3

Stefan's quintett by JWST

Turbulence injection in outflows

- SFR ~ 1 Msun/yr in whole galaxy

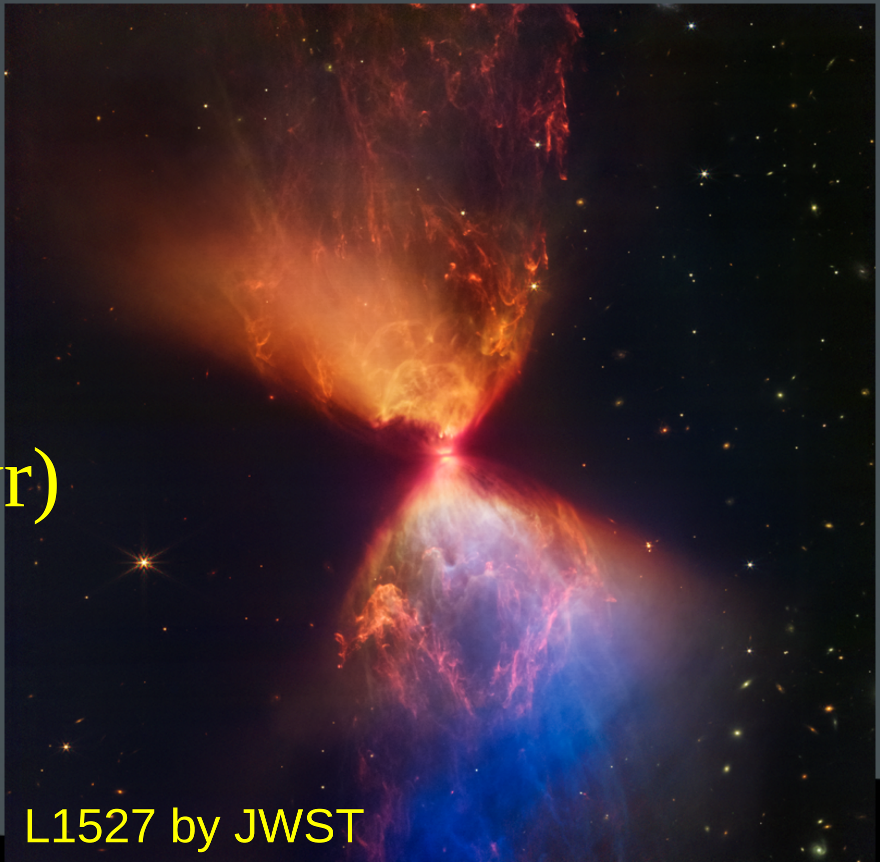
(volume 100pc.π.(10kpc)^2 ; density 1 /cm^3 =1Msun/pc^3)

- Assume 10% of stellar mass is fed back into ISM
- With velocity ~ 30 km/s

(free fall at 1 AU ~ 1st core radius for 1Msun)

→ Π = (30 km/s)^2/(10^6.3.10^5 yr)

= 6.10^{-8} cm^2/s^3


L1527 by JWST

Turbulence injection in stellar winds

- L/c momentum injection rate in radiation
- ~ 1-10% absorbed → launches winds
- Using 10% gas mass fraction and L/M=4 in solar units → momentum

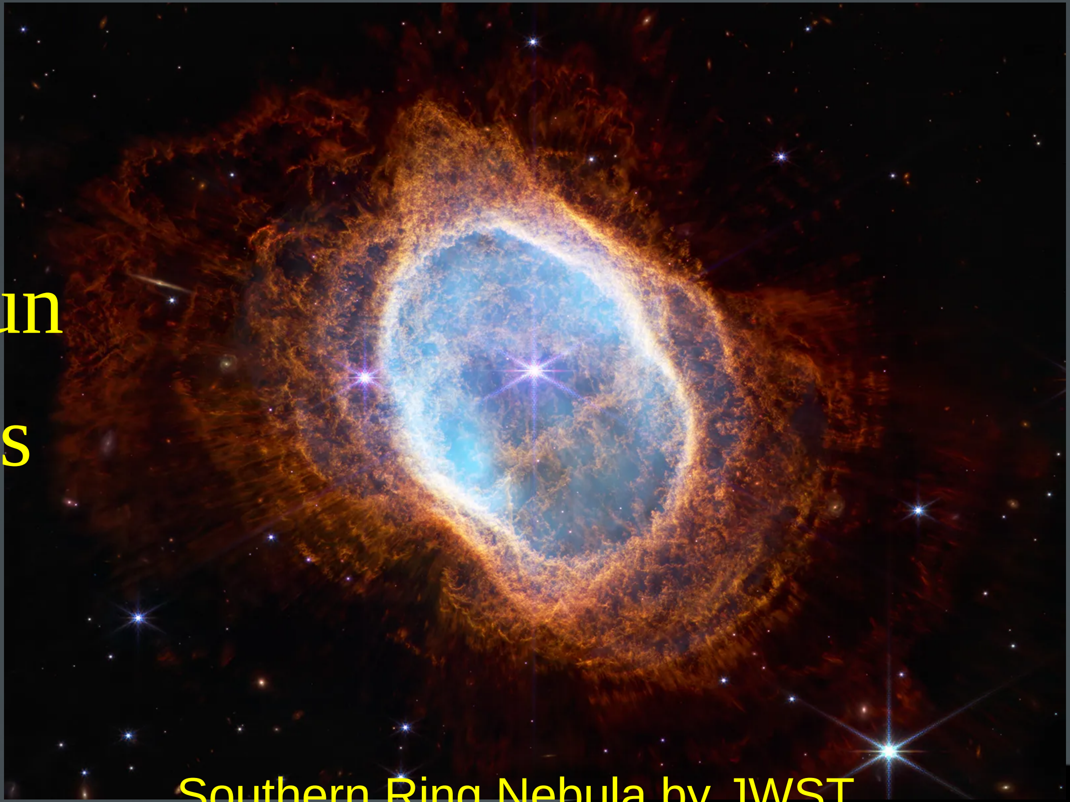
rate @ 100 pc is

10/100 Lsun/c for 10^6Msun

→ get ΔU/T, and find Π as

$$\Pi = R^{1/2}(\Delta U/T)^{3/2}$$

$$= 2.10^{-6}\ \mathrm{cm^2/s^3}$$

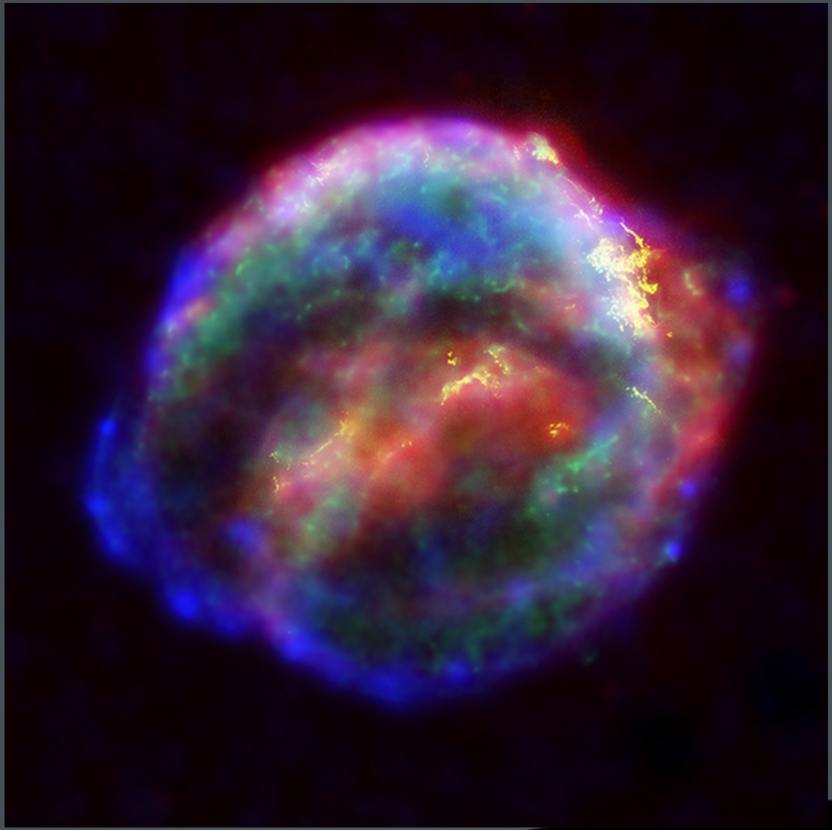


Southern Ring Nebula by JWST

Turbulence injection in SNe

- SNIa: 1.4 Msun / mC . 8 MeV = 1.7 10^{51} erg x 10% in kinetic energy → 10^{50} erg per SNIa.
- @100pc, ΔU~ 4 km/s
- ~1 SN(I+II)/25yr/galaxy

→ $\Pi \sim 0.005\ cm^2/s^3$



SN1604 by
Spitzer (R)
HST (G)
Chandra (B)

Ex: a termination shock (Zeta Oph)

Main problematics

- Shocks are ubiquitous in the interstellar medium, from the birth of stars to their death
- They convert kinetic energy into magnetic and thermal energy

=> They are excellent probes of the ISM dynamics

- They are a molecular and dust grains factory

Further open questions

- How much mass, momentum and energy is processed through interstellar shocks ?
- What is their role in the dissipation of large scale turbulent energy ?
- To what extent can we use the state-of-the-art and the upcoming tools to use shocks as probes of dynamics ?
- And probably many more questions...

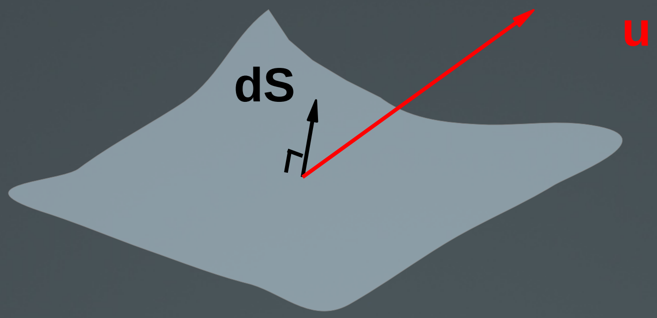
Hydrodynamics reminder

Conservation of

- Mass: continuity equation
- Momentum: Euler equation
- Energy
- Magnetic fields: induction equation

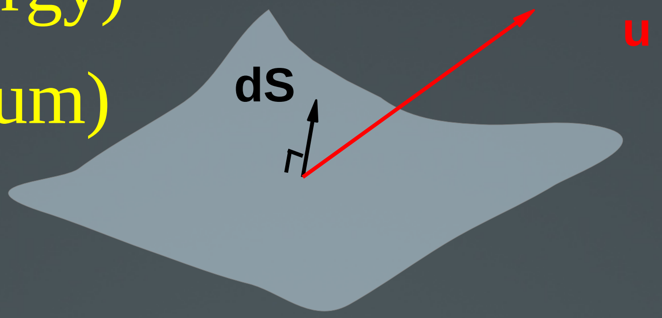
Flow / Flux

- Flux: [Quantity / Time / Surface]
  - mass flux= $\rho\ u_s$ (with ρ : mass density)
  - radiative flux
  - pressure
- Flow: [Quantity / Time]
  - mass flow= $\int \rho\ \mathbf{u_s.dS}$
  - luminosity



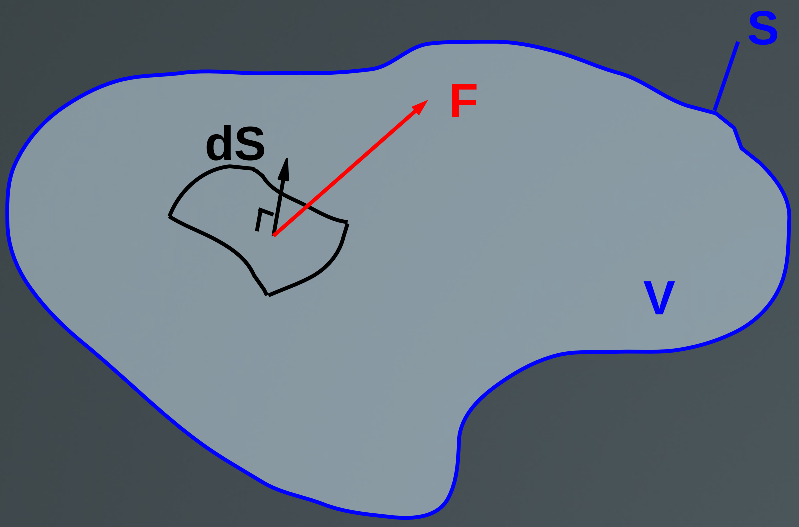
Flow / Flux

- Flux: [Quantity / Time / Surface]
  - mass flux= $\rho\ u_s$ (with ρ : mass density)
  - radiative flux (Quantity=energy)
  - pressure (Quantity=momentum)
- Flow: [Quantity / Time]
  - mass flow= $\int \rho\ \mathbf{u_s.dS}$
  - luminosity (Quantity=energy)



Conservation of Q

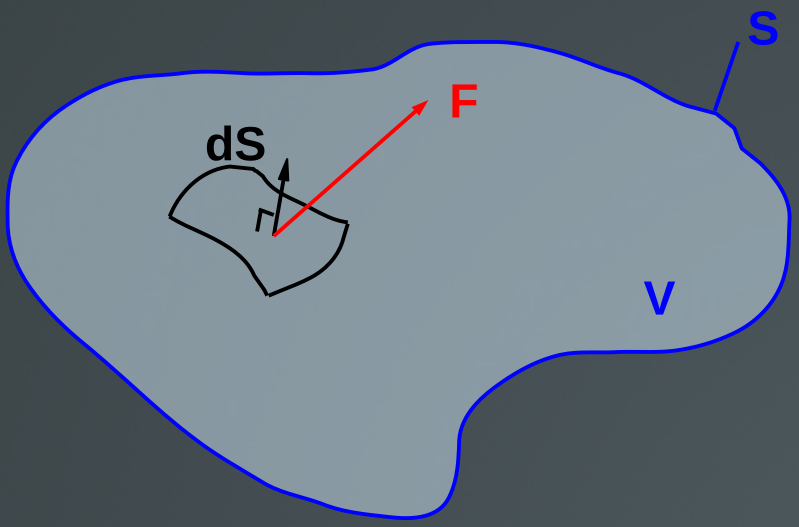
Time variation of Q in V + ∫ Flux of Q *out* of S = 0



$$\frac{\partial Q}{\partial t} + \oiint_S \boldsymbol{F_Q} . \boldsymbol{dS} = 0$$

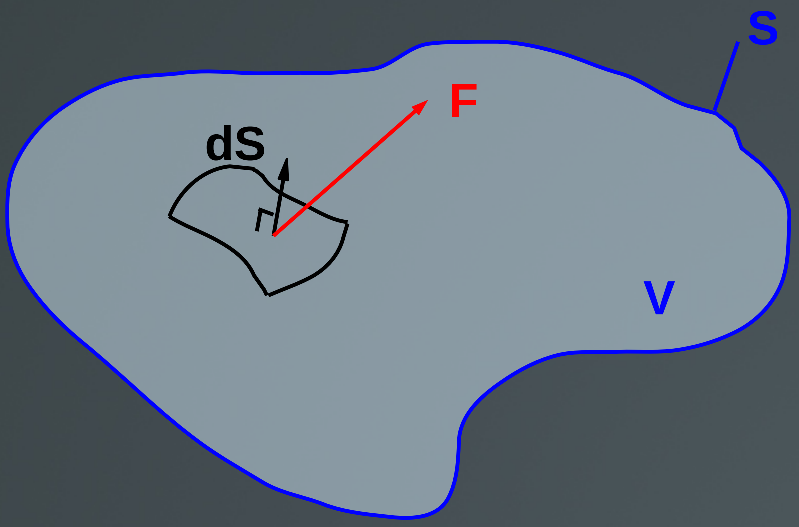
Ex: mass conservation

$$\frac{\partial M}{\partial t} + \oiint_S \rho \boldsymbol{u}.\boldsymbol{dS} = 0$$



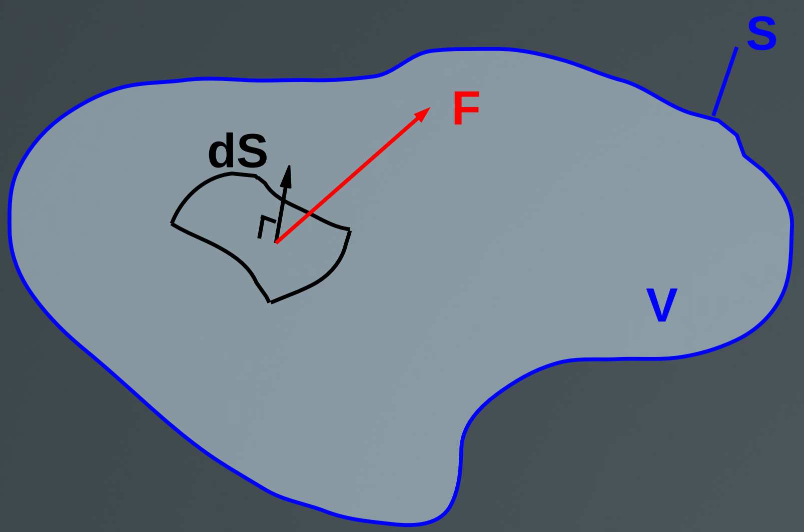
Ex: mass conservation

$$\frac{\partial}{\partial t}\iiint_V \rho dV + \oiint_S \rho \boldsymbol{u}.\boldsymbol{dS} = 0$$



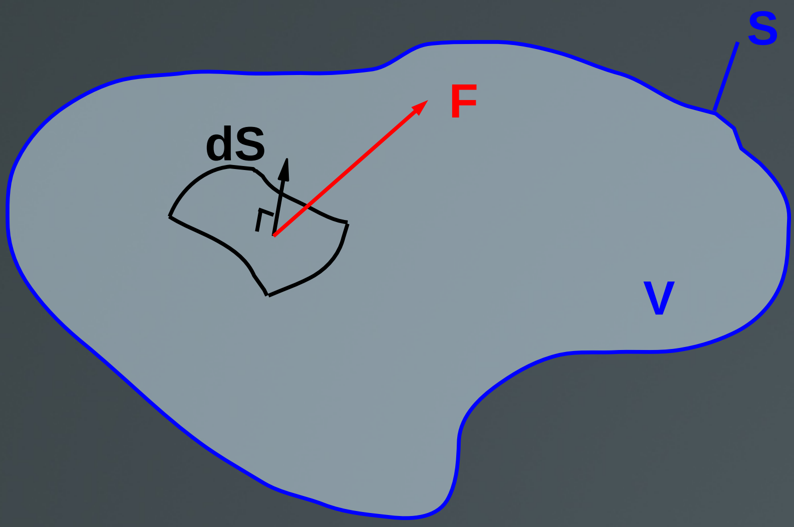
Green-Ostrogradski

$$\iiint_V (\nabla \cdot \mathbf{F})\, dV = \oiint_S (\mathbf{F} \cdot \mathbf{n})\, dS$$



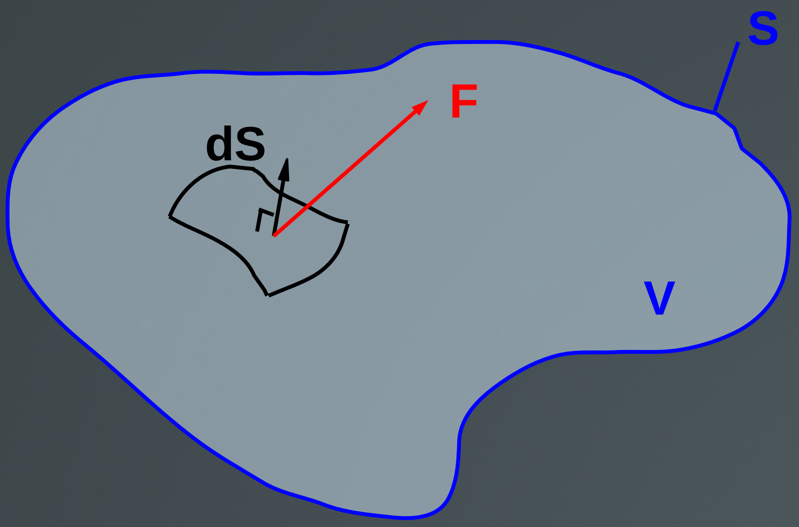
Ex: mass conservation

$$\frac{\partial}{\partial t}\iiint_V \rho dV + \iiint_V \boldsymbol{\nabla}.\rho u dV = 0$$



Ex: mass conservation

$$\iiint_V \left(\frac{\partial \rho}{\partial t} + \boldsymbol{\nabla}.\rho\boldsymbol{u}\right)dV = 0$$



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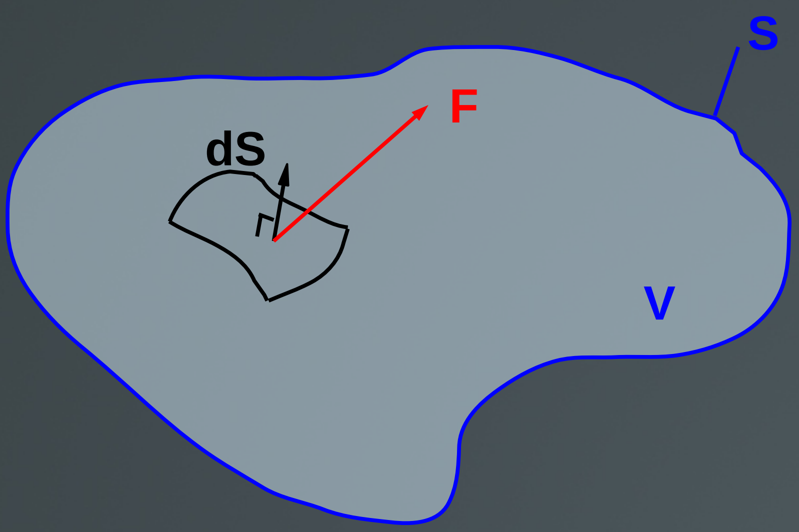
$$\iiint_V \left(\frac{\partial \rho}{\partial t} + \boldsymbol{\nabla}.\rho \boldsymbol{u}\right) dV = 0$$



$$\frac{\partial \rho}{\partial t} + \boldsymbol{\nabla}.(\rho \boldsymbol{u}) = 0$$

Isothermal ideal MHD equations

$$\frac{\partial \rho}{\partial t} + \boldsymbol{\nabla}.(\rho \boldsymbol{u}) = 0 \tag{1.1}$$

$$\partial_t \rho u_i + \partial_j (\rho u_i u_j + p\delta_{ij}) - \boldsymbol{J} \times \boldsymbol{B} = 0 \tag{1.2}$$

$$\partial_t \boldsymbol{B} - \boldsymbol{\nabla} \times (\boldsymbol{u} \times \boldsymbol{B}) = 0 \tag{1.3}$$

with $p(\rho)$ given, for example, isothermal EoS: $p = \rho c^2$
and with the current vector $\boldsymbol{J} = \frac{1}{4\pi} \boldsymbol{\nabla} \times \boldsymbol{B}$

Global form for ideal fluids:

$$\partial_t \boldsymbol{W} + \boldsymbol{\nabla}.\boldsymbol{\mathcal{F}}(\boldsymbol{W}) = 0$$

where $\boldsymbol{W}$ contains all fluid states variables: $\boldsymbol{W} \equiv (\rho, \rho \boldsymbol{u}, \boldsymbol{B})$

Non-ideal isothermal MHD equations

$$\frac{\partial \rho}{\partial t} + \boldsymbol{\nabla}.(\rho \boldsymbol{u}) = 0 \quad (1.5)$$

$$\partial_t \rho u_i + \partial_j(\rho u_i u_j + p\delta_{ij} - \nu\rho S_{ij}[u]) - \boldsymbol{J} \times \boldsymbol{B} = 0. \quad (1.6)$$

$$\partial_t \boldsymbol{B} - \boldsymbol{\nabla}\times \left(\boldsymbol{u} \times \boldsymbol{B} + \frac{1}{F_{in}}(\boldsymbol{J} \times \boldsymbol{B}) \times \boldsymbol{B} - \eta \boldsymbol{\nabla} \times \boldsymbol{B} \right) = 0. \quad (1.7)$$

With the viscous stresses:

$$S_{ij}[u] = \frac{1}{2}(\partial_i u_j + \partial_j u_i) - \frac{1}{3}\partial_k u_k \delta_{ij} \quad (1.8)$$

And the ion-neutral momentum exchange rate:

$$F_{in} = \rho_c \rho \left\langle \sigma v \right\rangle_{in} / (\mu_c + \mu_n) \quad (1.9)$$

Global form with a diffusive flux $\boldsymbol{\mathcal{F}}_d$:

$$\partial_t \boldsymbol{W} + \boldsymbol{\nabla}.[\boldsymbol{\mathcal{F}}(\boldsymbol{W}) + \boldsymbol{\mathcal{F}}_d(\boldsymbol{W}, \partial_i \boldsymbol{W})] = 0$$

Waves

Planar wave definition

The fluid state variables on a planar wave depend on a single coordinate, and advance at a constant speed c

$$\boldsymbol{W}(x - ct) \tag{2.1}$$

Plane waves therefore ‘carry information’ at speed c

Ideal planar wave

$$\boldsymbol{W}(x - ct) \tag{2.1}$$

Plug that form in the global form

$$\partial_t \boldsymbol{W} + \boldsymbol{\nabla}.\boldsymbol{\mathcal{F}}(\boldsymbol{W}) = 0 \tag{2.2}$$

To obtain:

$$-c\,\partial_x \boldsymbol{W} + \frac{\partial \boldsymbol{\mathcal{F}}_x}{\partial \boldsymbol{W}}.\partial_x \boldsymbol{W} = 0 \tag{2.3}$$

$\Rightarrow \partial_x \boldsymbol{W}$ is an eigenvector of $\frac{\partial \boldsymbol{\mathcal{F}}_x}{\partial \boldsymbol{W}}$, associated to the eigenvalue c.

The eigenvectors usually form an orthogonal basis

(the system is said *hyperbolic* when all eigenvalues are different)

A 'feature' at a given spot can be decomposed in this basis: each component is transported at its own speed.

Ideal planar wave

$$\boldsymbol{W}(x - ct) \tag{2.1}$$

Plug that form in the global form

$$\partial_t \boldsymbol{W} + \boldsymbol{\nabla}.\boldsymbol{\mathcal{F}}(\boldsymbol{W}) = 0 \tag{2.2}$$

To obtain:

$$-c\,\partial_x \boldsymbol{W} + \frac{\partial \boldsymbol{\mathcal{F}}_x}{\partial \boldsymbol{W}}.\partial_x \boldsymbol{W} = 0 \tag{2.3}$$

$\Rightarrow \partial_x \boldsymbol{W}$ is an eigenvector of $\frac{\partial \boldsymbol{\mathcal{F}}_x}{\partial \boldsymbol{W}}$, associated to the eigenvalue c.

Galilean invariance:

The equations are invariant under Galilean transformations:
$u_x \to u_x + u_0$ implies $c \to c + u_0$

Linear ideal wave

$$\partial_t \boldsymbol{W} + \boldsymbol{\nabla}.\boldsymbol{\mathcal{F}}(\boldsymbol{W}) = 0$$

Assume a homogeneous background $\boldsymbol{W}_0$ with a small perturbation $\boldsymbol{W}_1$:

$$\boldsymbol{W} = \boldsymbol{W}_0 + \boldsymbol{W}_1 \tag{2.8}$$

And prescribe a complex form $\boldsymbol{W}_1 = \delta\boldsymbol{W} \exp(i\boldsymbol{k}.\boldsymbol{r} - i\omega t)$
You arrive at

$$-i\omega\delta\boldsymbol{W} + ik\frac{\partial(\boldsymbol{\mathcal{F}}.\hat{\boldsymbol{k}})}{\partial\boldsymbol{W}}.\delta\boldsymbol{W} = 0 \tag{2.9}$$

$$-c\,\partial_x\boldsymbol{W} + \frac{\partial\boldsymbol{\mathcal{F}}_x}{\partial\boldsymbol{W}}.\partial_x\boldsymbol{W} = 0$$

$$\frac{\omega}{k} \longleftrightarrow c$$

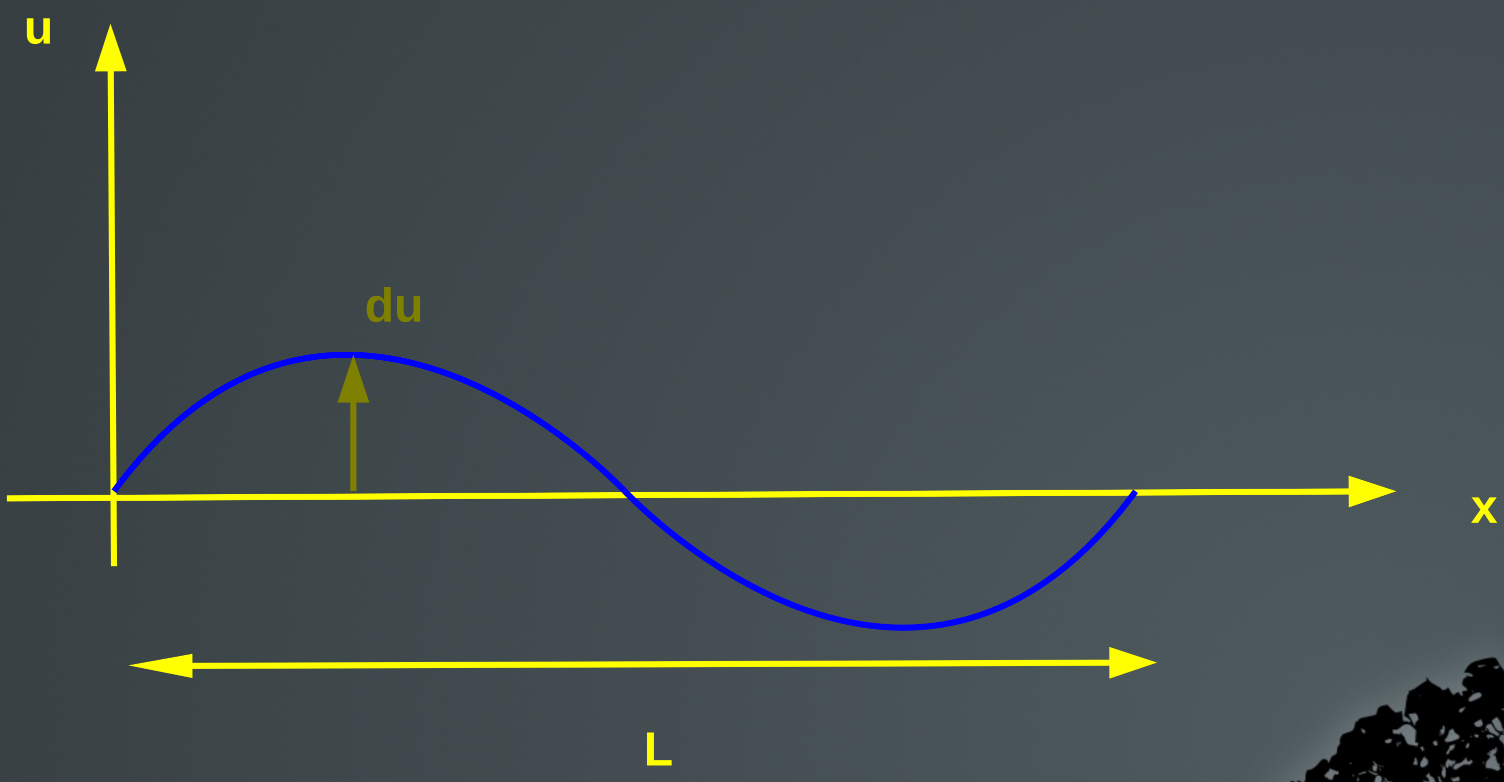
$$\delta\boldsymbol{W} \longleftrightarrow \partial_x\boldsymbol{W}$$

Ex: Barotropic hydrodynamics

$$c = \sqrt{\frac{\partial P}{\partial \rho}}$$

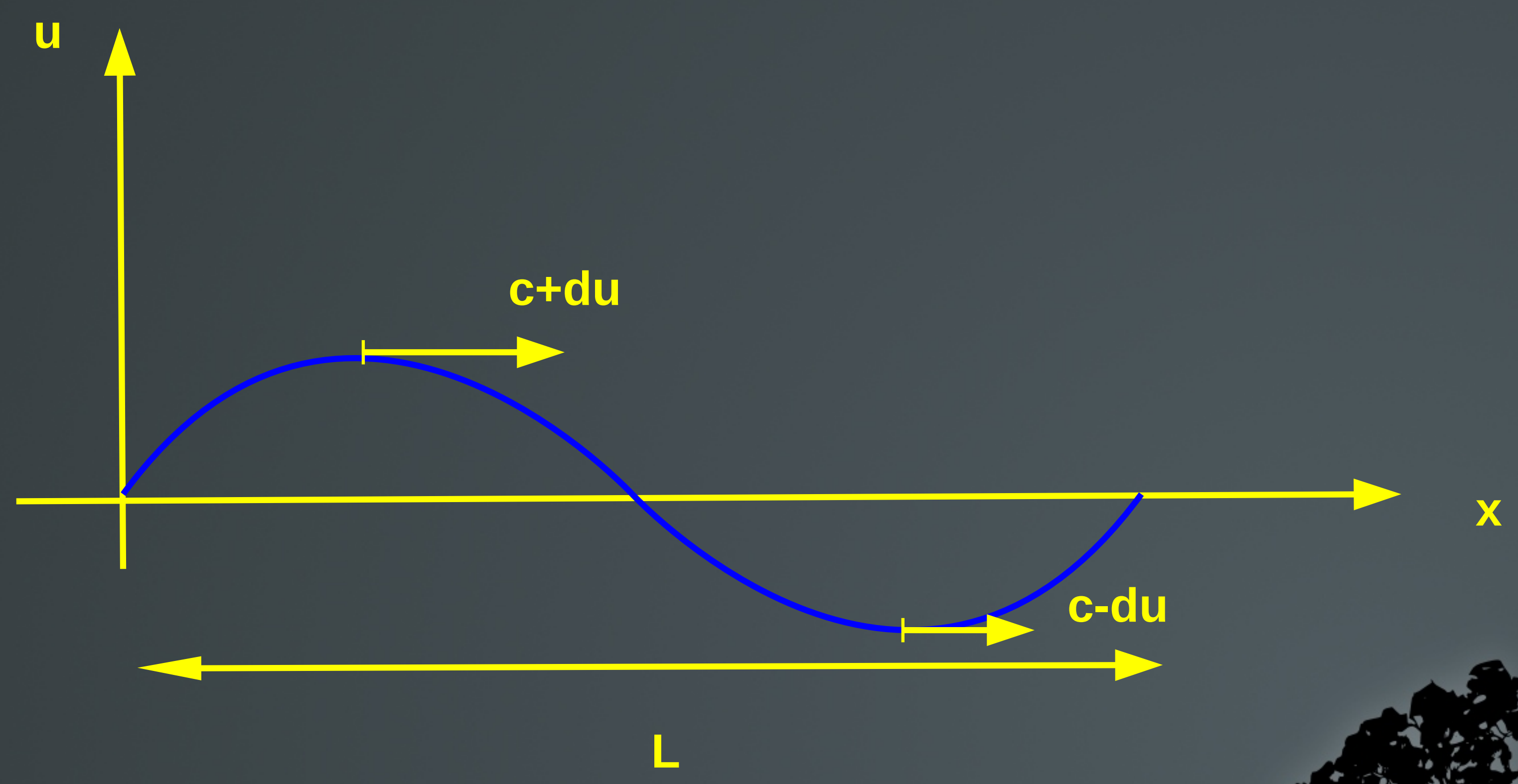
Wave steepening

- Take a compression wave:



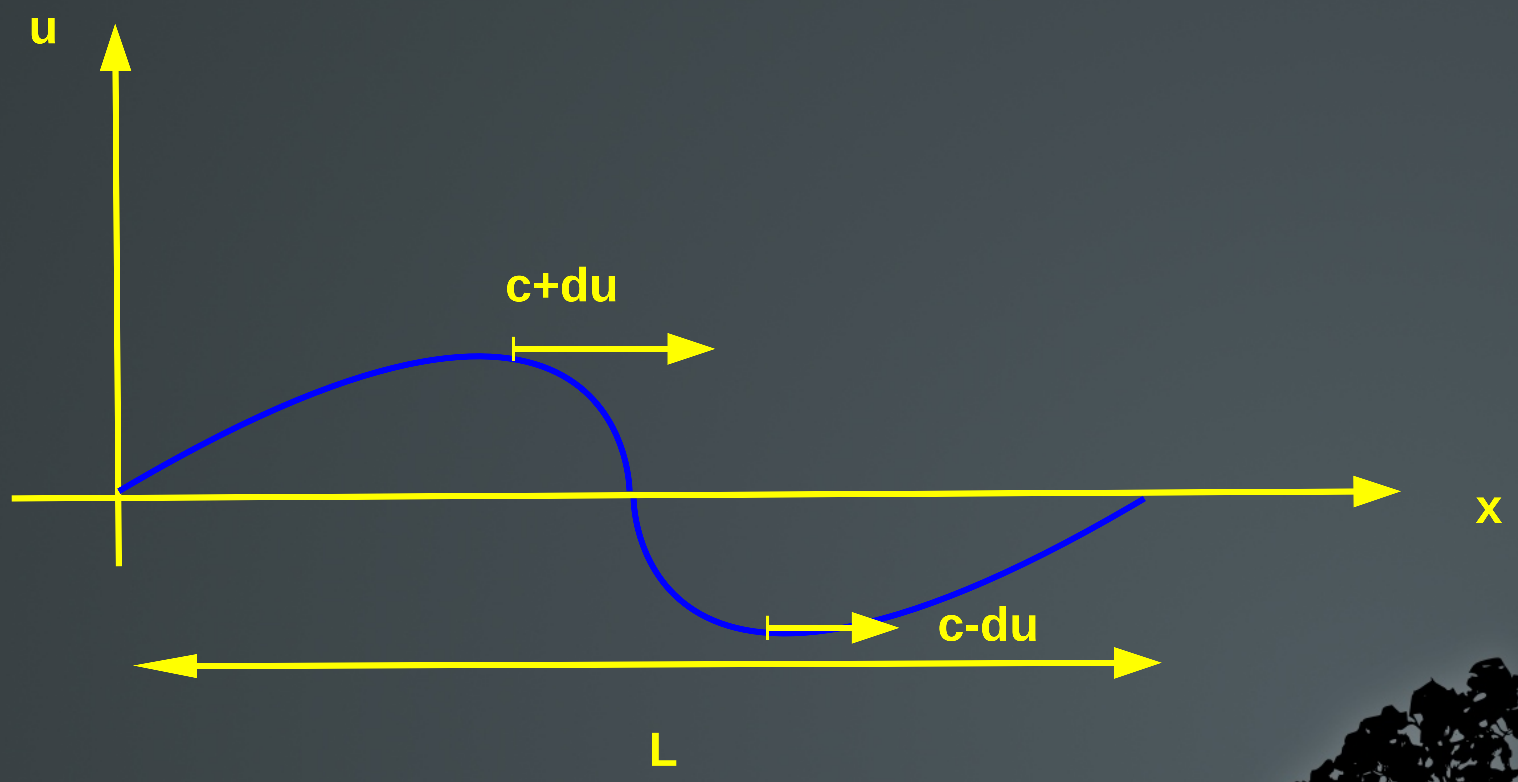
Wave steepening

- Left bump catches up with right trough:



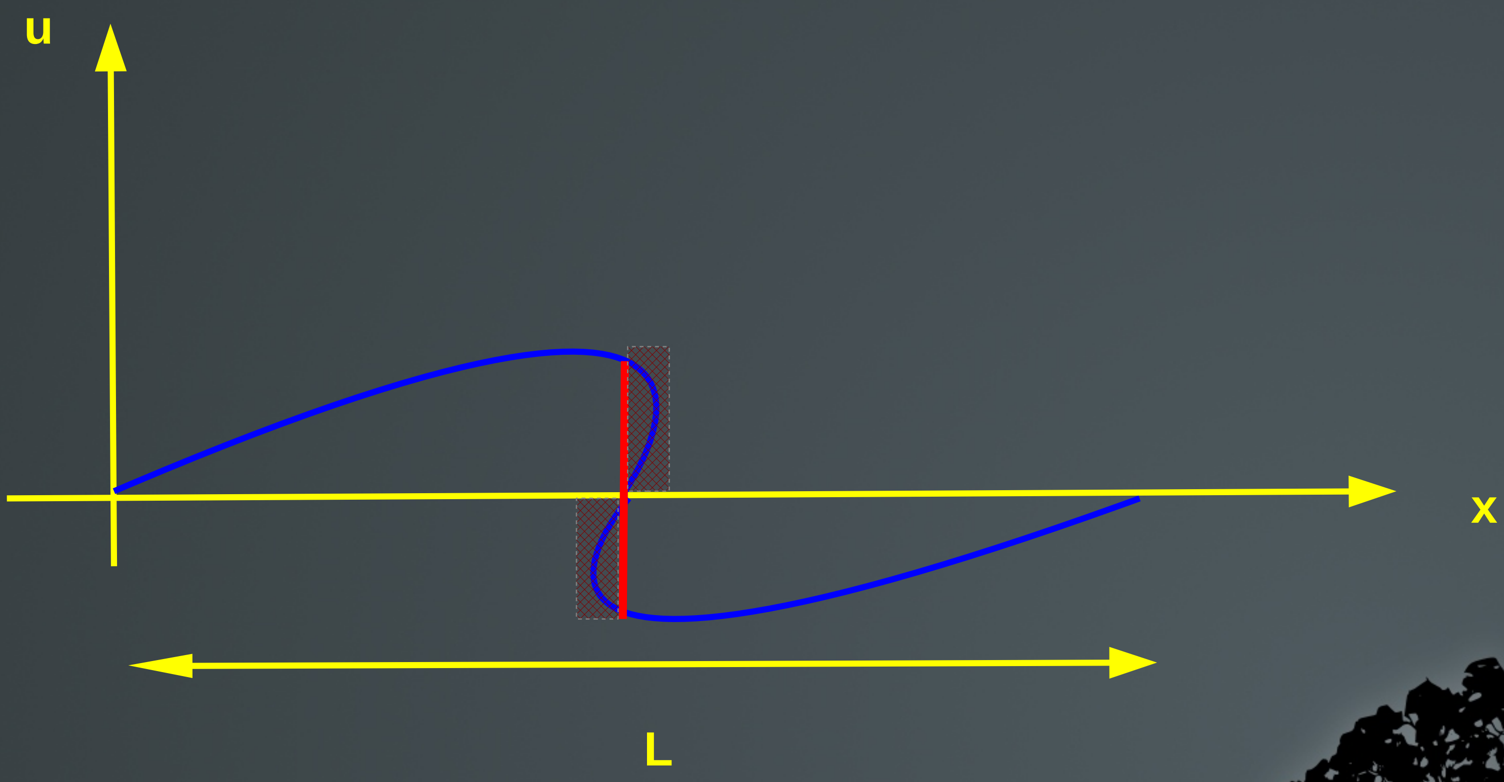
Wave steepening

- Wave steepens:



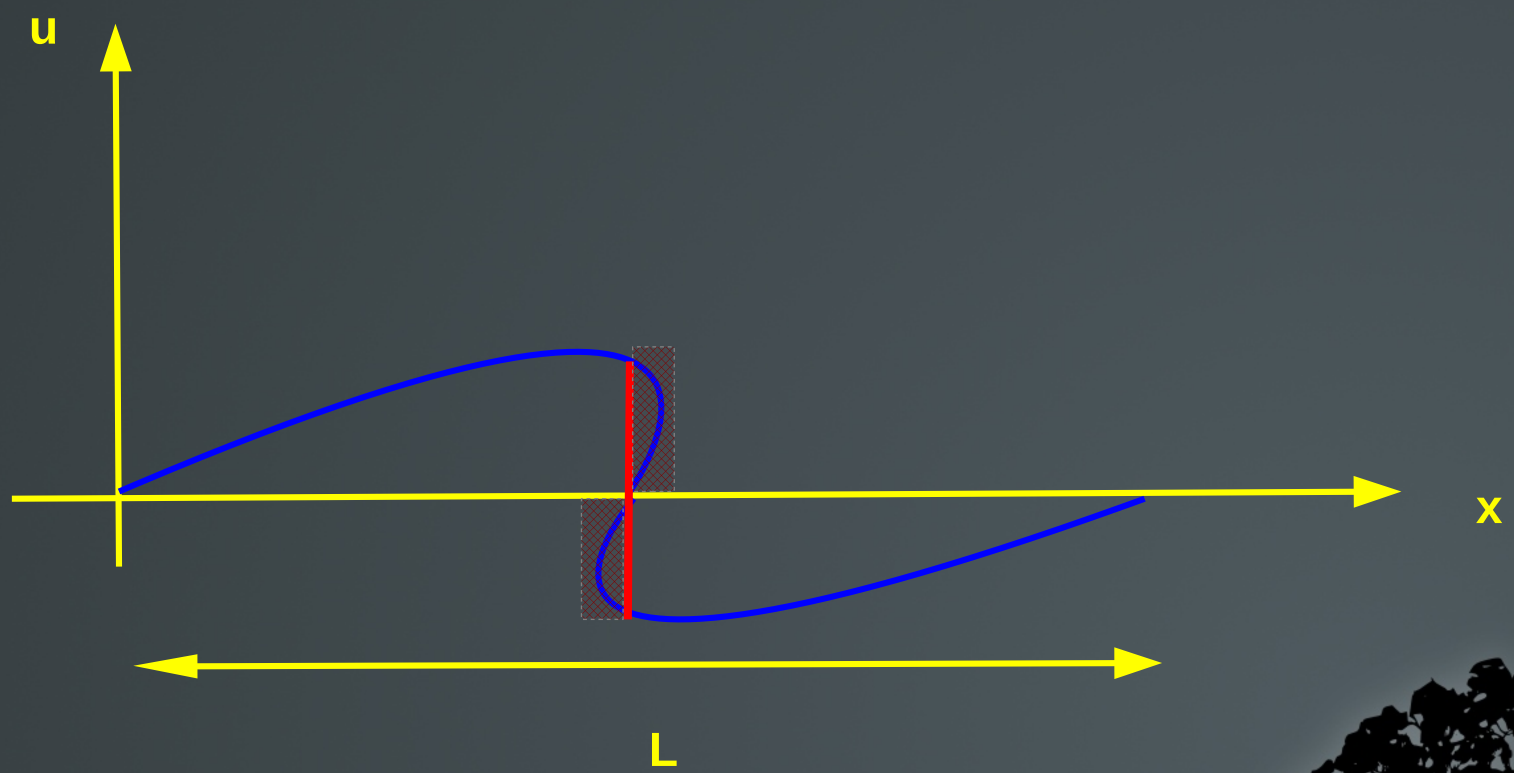
Wave steepening

- A shock is born !



Wave steepening

- A shock is born ! It takes a time ~ ¼ L/du ~ ¼ T c/du
- A variation of c(x) can also lead to shell crossing



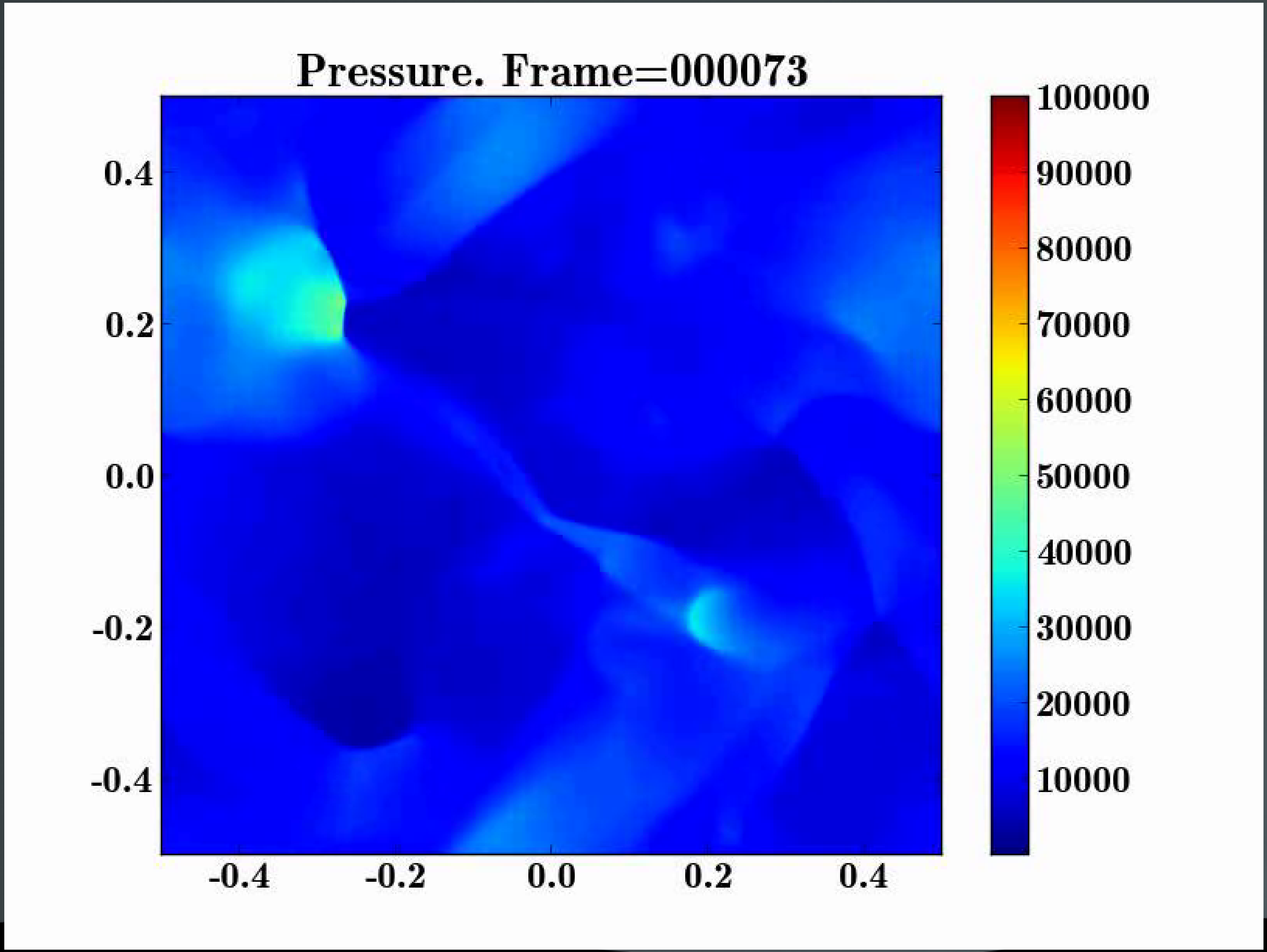
Steepening of a surface wave

Ex: Barotropic hydrodynamics

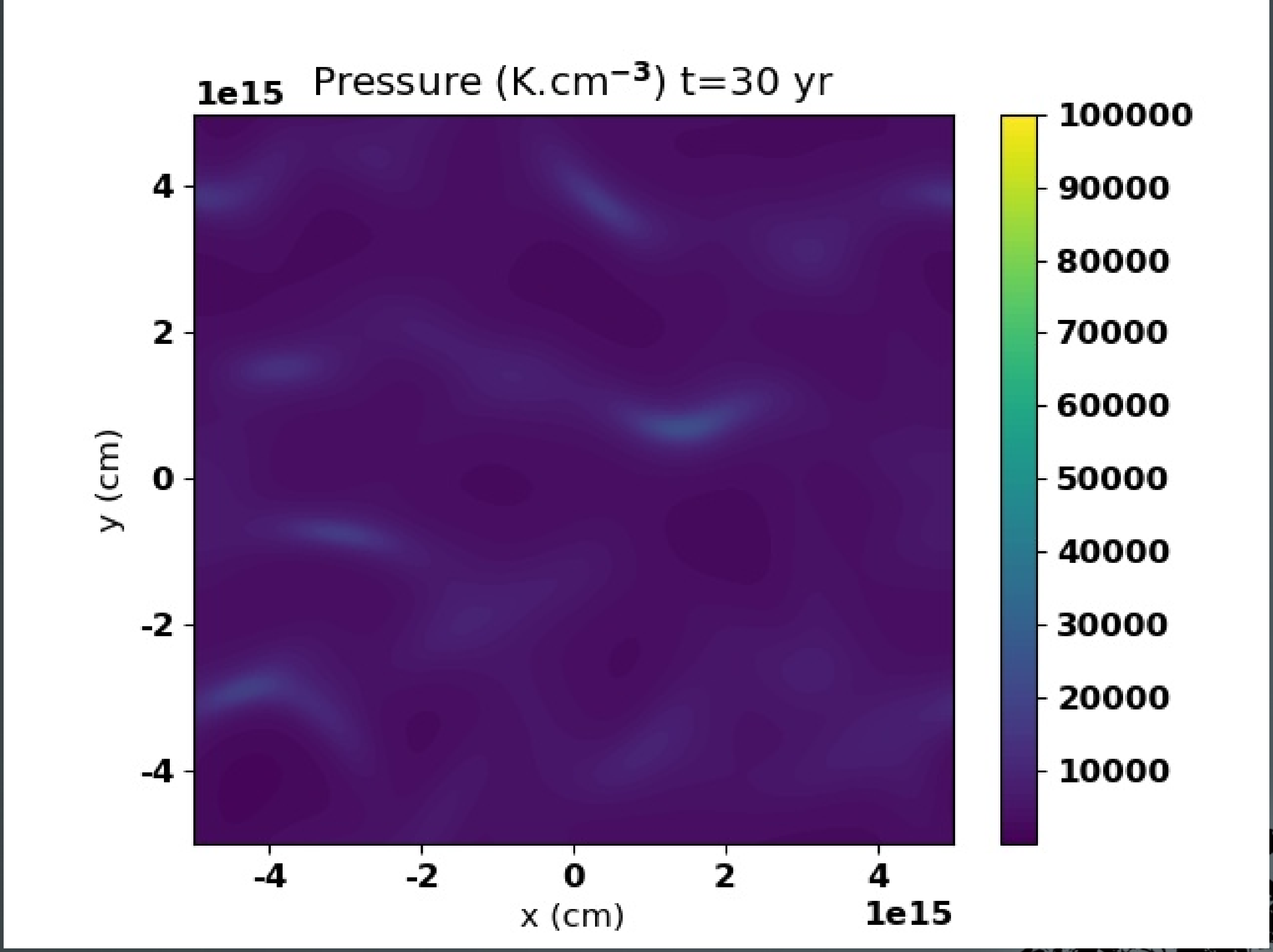
$$c = \sqrt{\frac{\partial P}{\partial \rho}}$$

Shocks in decaying 2D turbulence

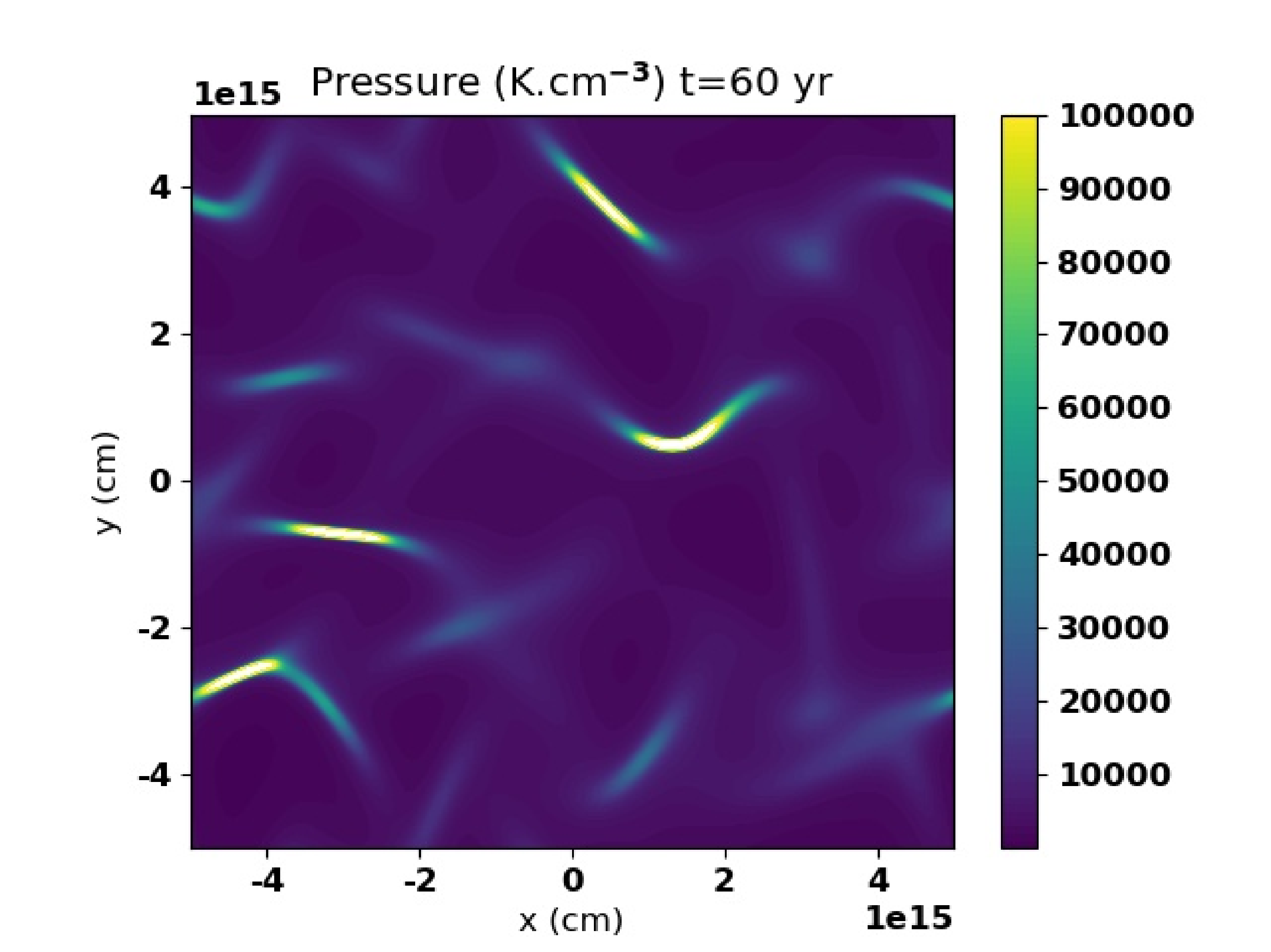
(Lesaffre+2020)



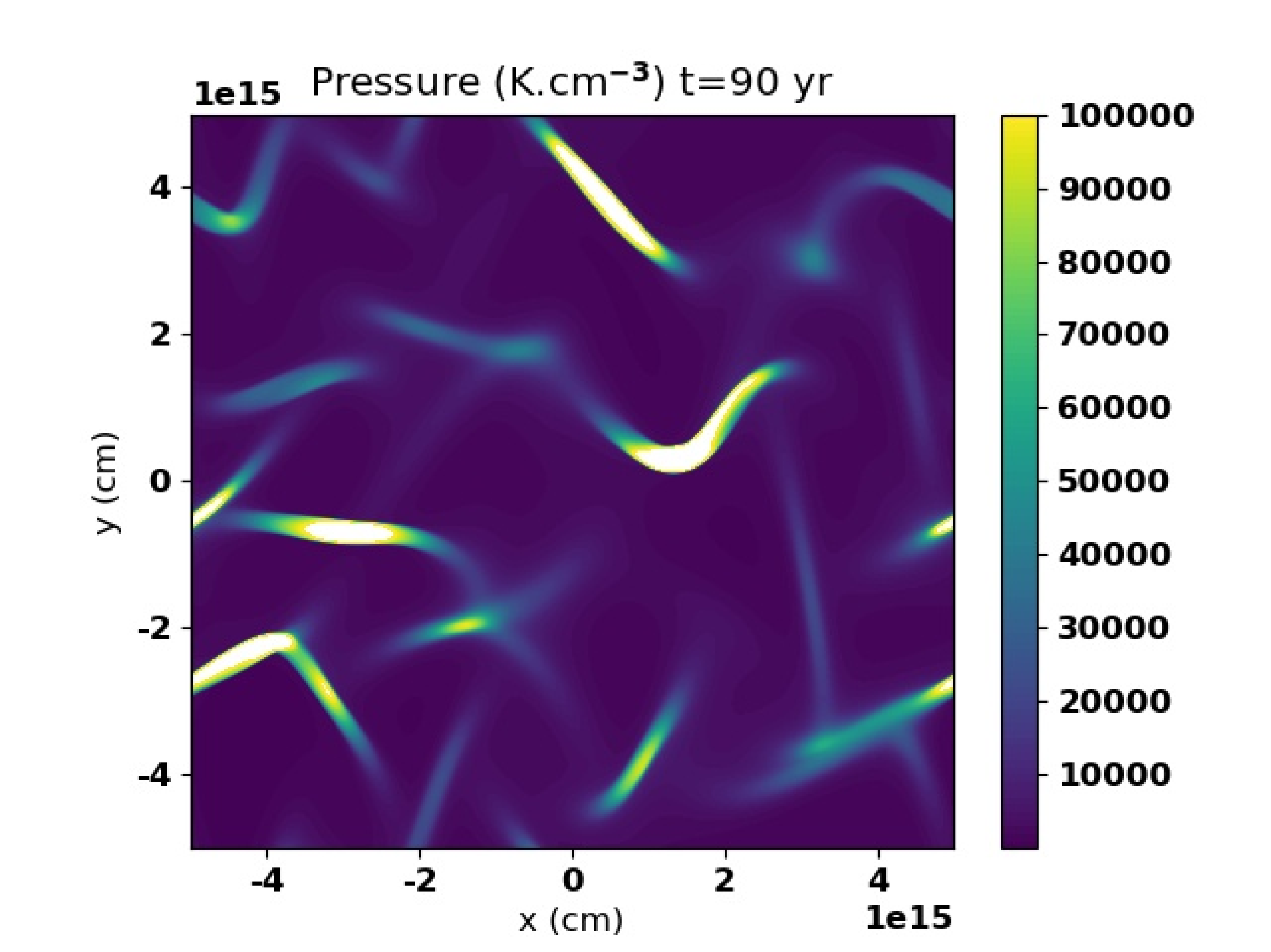
Formation of shocks in 2D



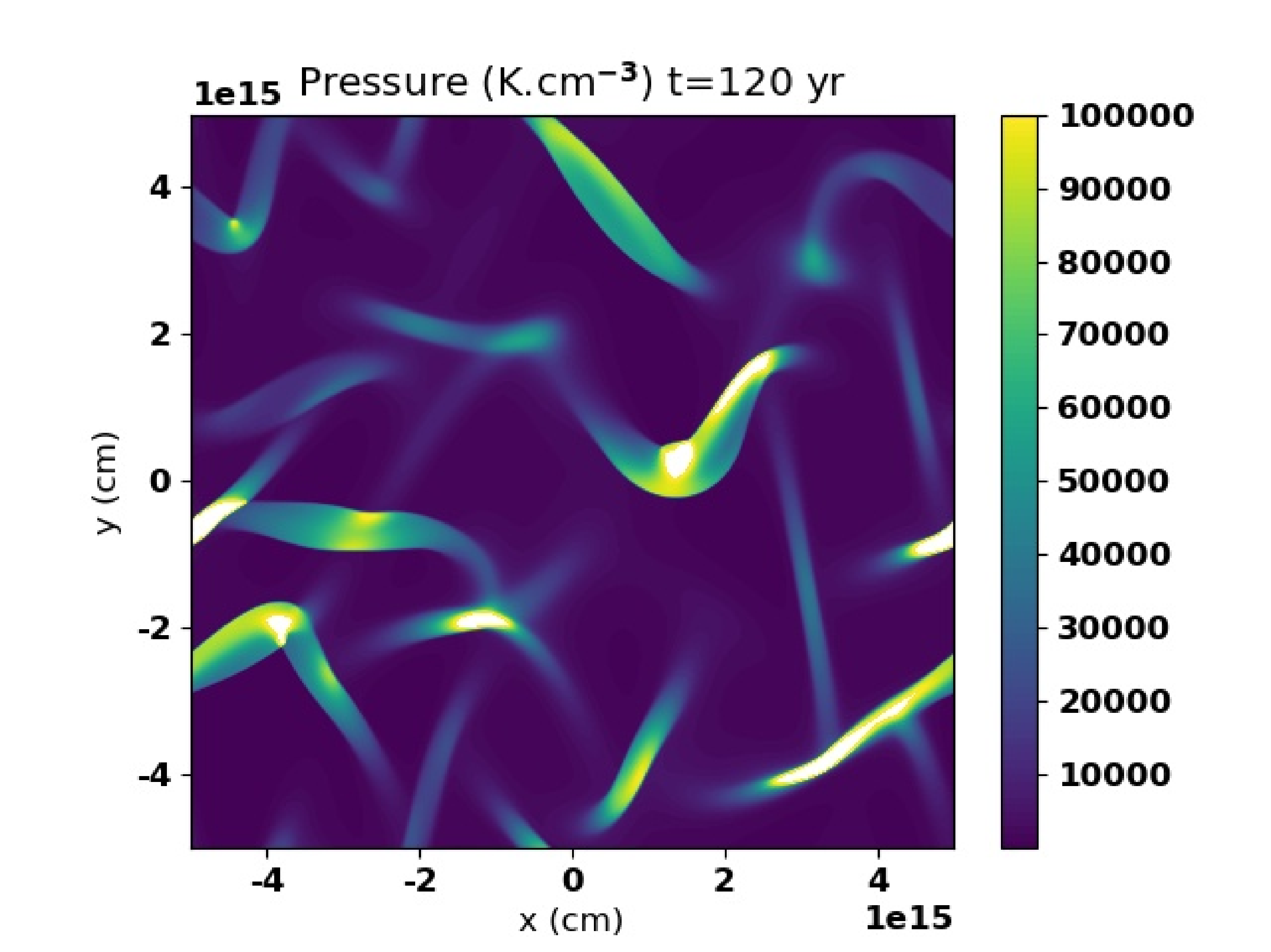
Formation of shocks in 2D



Formation of shocks in 2D

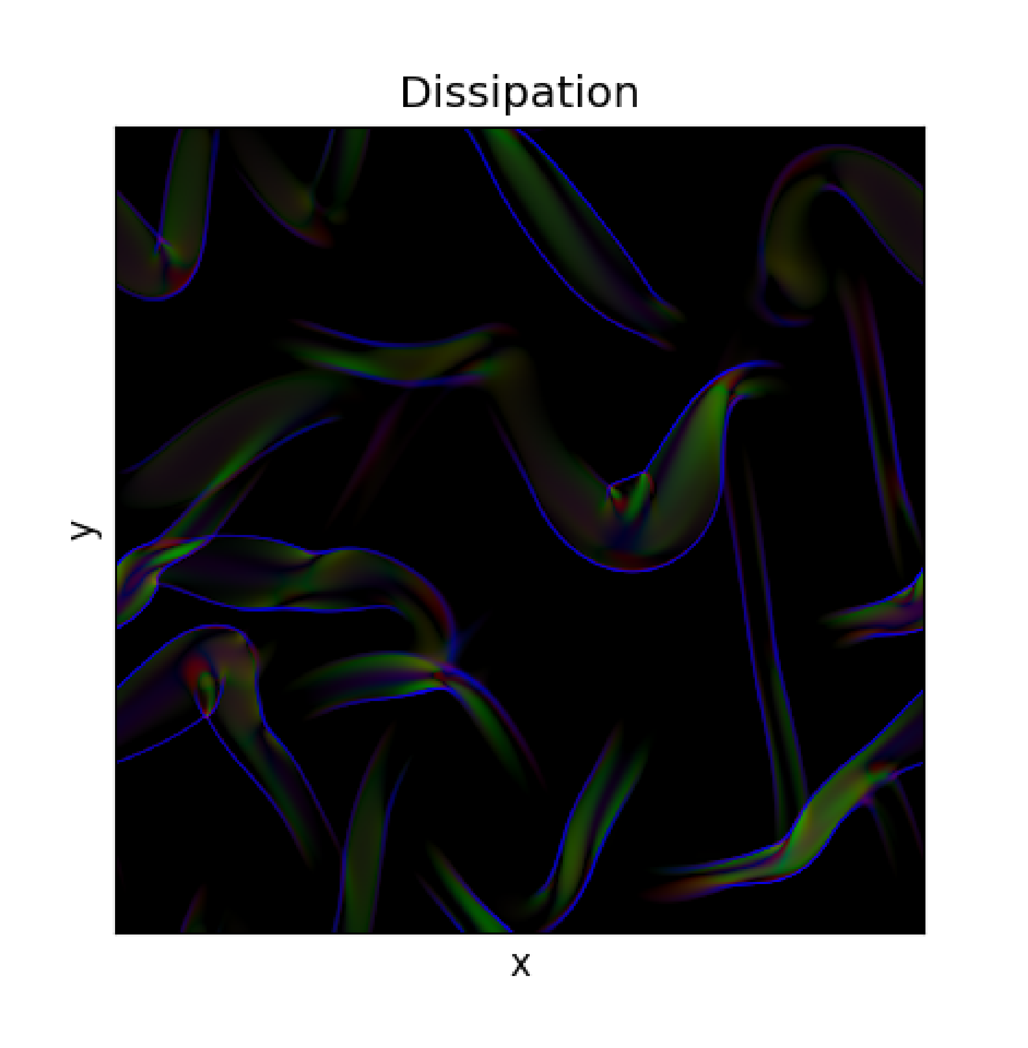


Formation of shocks in 2D



Formation of shocks in 2D

Shocks form back to back with a shear layer in between

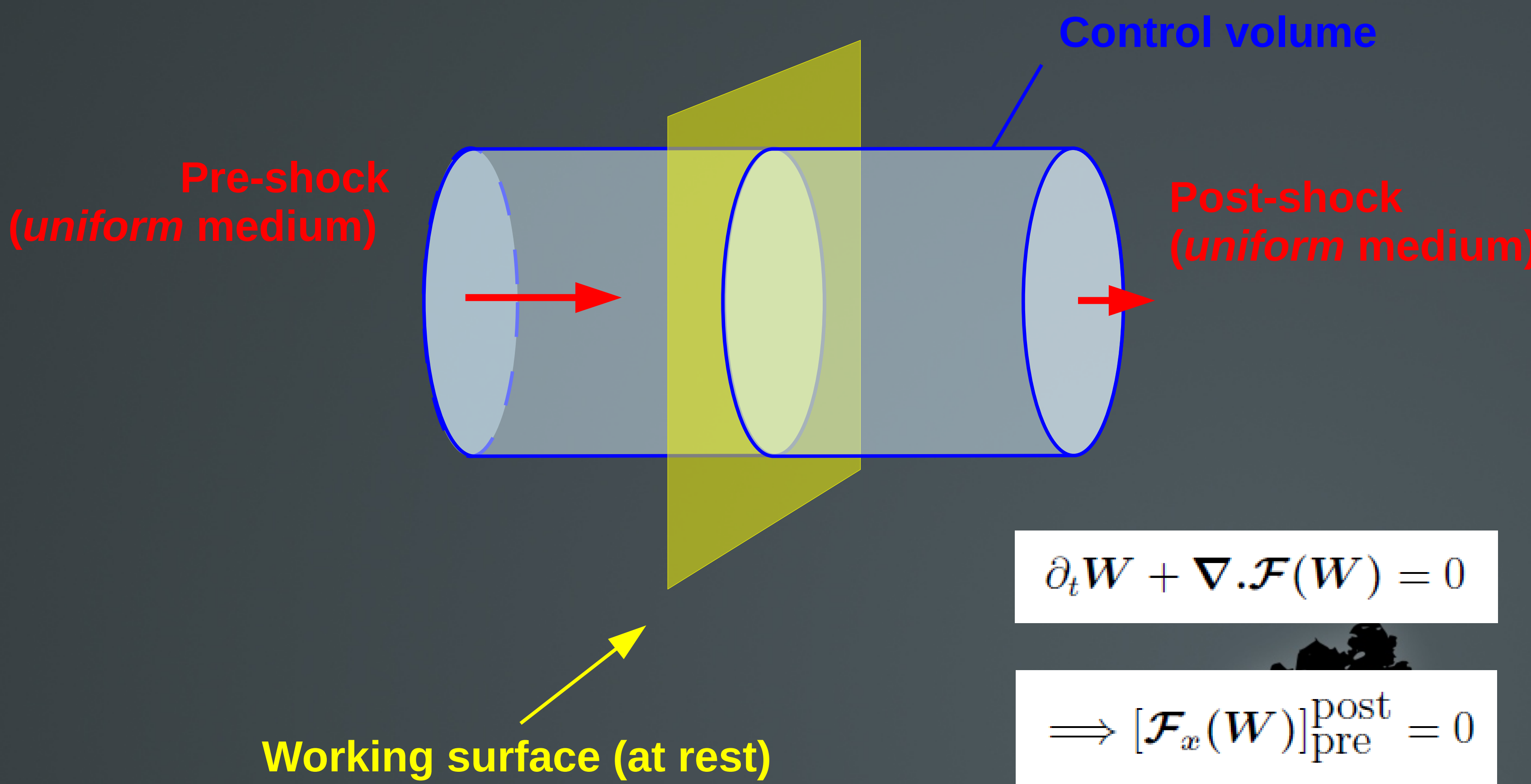


Blue: compressive heating
Green: vortical heating

Shock wave
Jump conditions (Rankine-Hugoniot)

Rankine Hugoniot relations

- Flux conservation through a *steady planar* shock



$$\partial_t W + \boldsymbol{\nabla}.\boldsymbol{\mathcal{F}}(W) = 0$$

$$\Longrightarrow [\boldsymbol{\mathcal{F}}_x(W)]_{\text{pre}}^{\text{post}} = 0$$

Rankine Hugoniot

- Conservation of mass, momentum and magnetic flux *in the steady shock frame* induces relationships between pre-shock and post-shock physical conditions.

$$\left[B_x\right]_{pre}^{post} = 0$$

$$\left[\rho\, u_x\right]_{pre}^{post} = 0$$

$$\left[(B \times u)_y\right]_{pre}^{post} = 0$$

$$\left[(B \times u)_z\right]_{pre}^{post} = 0$$

$$[\rho u_x^2 + P + \frac{B^2}{8\pi}]_{pre}^{post} = 0$$

$$[\rho u_x u_y - \frac{B_x B_y}{4\pi}]_{pre}^{post} = 0$$

$$[\rho u_x u_z - \frac{B_x B_z}{4\pi}]_{pre}^{post} = 0$$

- <u>Examples:</u>

* Compression = Mach^2 in an isothermal shock
* Max temperature ~ u^2 expressses conversion of kinetic to thermal energy in a viscous front

For the molecular weight of the ISM: $T_{\max} = 53\,\mathrm{K}\,(u/1\,\mathrm{km\ s^{-1}})^2$

Classification of MHD discontinuities

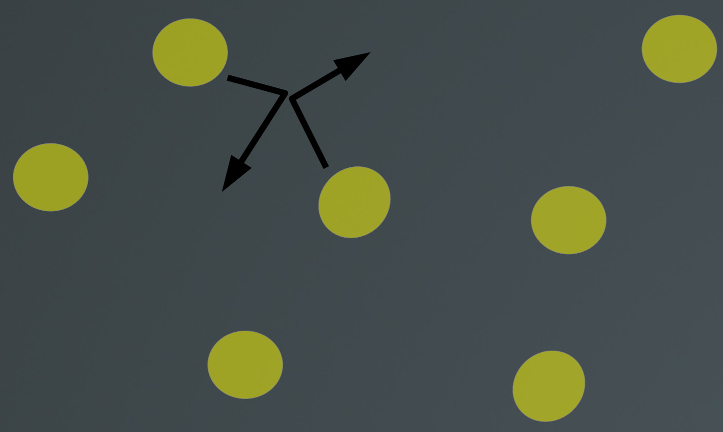
	$[u_x] = 0$	$[u_x]$ non zero
$[\rho] = 0$	(uniform solution)	Rotational Discontinuities
$[\rho]$ non zero	Contact Discontinuities	***Shocks***

Shocks are further classified according to
the variation of the transverse component of B:
- it changes sign in ***intermediate shocks***
- it grows in ***fast shocks***
- it decreases in ***slow shocks***

Internal shock structure and dissipation

Viscous dissipation
neutral – neutral collisions

- Collisions transfer momentum



ν = mean free path x thermal velocity

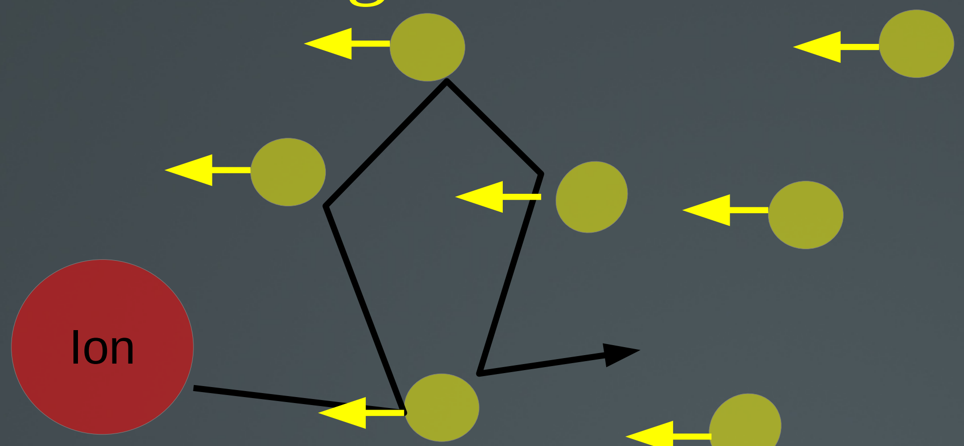
- Viscous pressure: $\pi = $ (4/3 factor if u_x) $\rho\ \nu\ d(u)/dx$
- Viscous spread of a shock: $\ell = \nu\ /\ u$

Ambipolar diffusion
ions collisions on neutrals

- Two-fluids: separate momenta for charges & neutrals
- Angular momentum transfer through ion-neutral collisions: ion-neutral drag

$$1.9 \times 10^{-9} \mathrm{cm}^3 \mathrm{s}^{-1}$$

$$F_{in} = \rho_i \rho \frac{\langle \sigma v \rangle_{in}}{\mu_i \mu_n} (u_n - u_i)$$



- Ambipolar diffusion: assume ion inertia is negligible

→ balance between friction and Lorentz-force

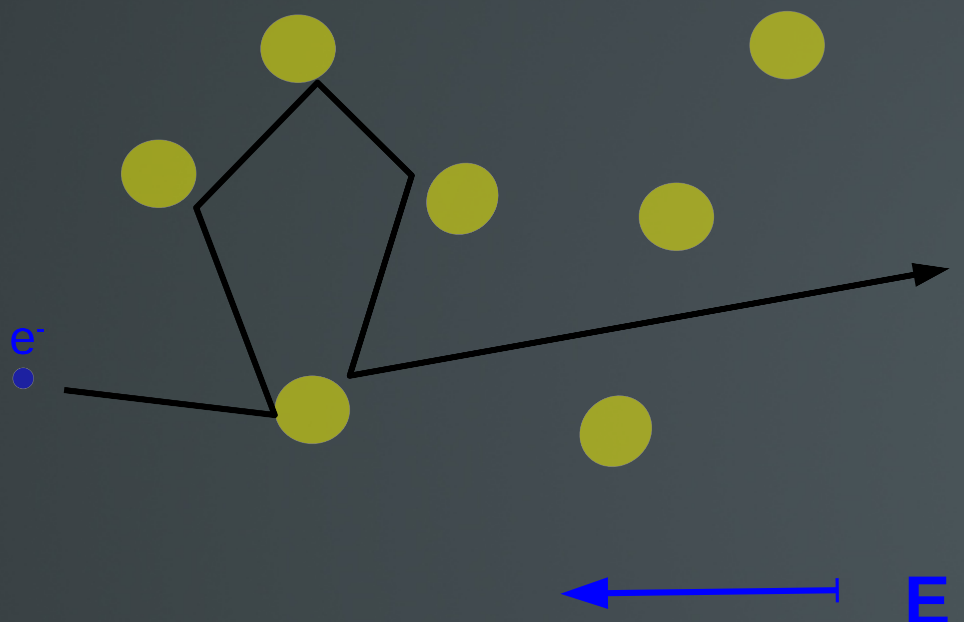
→ neutrals feel the Lorentz force $J \times B = F_{in}$

Resistivity
e- collisions on neutrals

- Impulsion drift acquired between 2 collisions: $E\tau = m_e u$
- Charge drift current: $J = n_e u$
- Resistivity: $\eta \sim \frac{E}{J} = \frac{m_e}{n_e \tau} = \frac{m_e v_e^{th}}{n_e \lambda_e}$

$$\eta = 234 \frac{n}{n_e} \sqrt{T[K]} \mathrm{cm^2 s^{-1}}$$

(Balbus & Terquem 2001)



Internal shock structure

- Consider dissipation to connect the two uniform states:

In the steady-state frame $\partial_t \equiv 0$, hence:

$$\partial_x[\boldsymbol{\mathcal{F}}(\boldsymbol{W}) + (\eta \text{ or } \nu) \text{ stress}] = 0$$

$$\boldsymbol{\mathcal{F}}(\boldsymbol{W}) + (\eta \text{ or } \nu)\partial_x(\text{ some component of } u \text{ or } B) = \boldsymbol{\mathcal{C}}$$

where $\boldsymbol{\mathcal{C}}$ is the constant vector of conserved fluxes.

=> we arrive at a set of algebraic differential equations, which can usually be expressed as ODEs.

=> Shocks can be parametrised by their conserved fluxes

Time-dependent MHD shocks equations:

Complicated...

Two partial derivatives: time and space

Hard to solve:
prone to numerical instabilities and
large CPU cost (few hours for 32 chemical species)

$$\frac{\partial}{\partial t}(n_j) + \frac{\partial}{\partial x}(n_j u_\mathrm{n} + J_j) = R_j \quad \text{for } j \text{ neutral specie} \tag{1}$$

$$\frac{\partial}{\partial t}(n_j) + \frac{\partial}{\partial x}(n_j u_\mathrm{c} + J_j) = R_j \quad \text{for } j \text{ ionic specie} \tag{2}$$

$$\frac{\partial}{\partial t}(\rho_\mathrm{n} u_\mathrm{n}) + \frac{\partial}{\partial x}(\rho_\mathrm{n} u_\mathrm{n}^2 + p_\mathrm{n} + \pi_\mathrm{n}) = F_{\mathrm{c}\to\mathrm{n}} \tag{3}$$

$$\frac{\partial}{\partial t}(\rho_\mathrm{c} u_\mathrm{c}) + \frac{\partial}{\partial x}\left(\rho_\mathrm{c} u_\mathrm{c}^2 + p_\mathrm{c} + \pi_\mathrm{i} + \frac{B^2}{8\pi}\right) = F_{\mathrm{n}\to\mathrm{c}} \tag{4}$$

$$\frac{\partial}{\partial t}\left(\frac{1}{\gamma-1}p_\mathrm{n} + \frac{1}{2}\rho_\mathrm{n} u_\mathrm{n}^2\right) + \frac{\partial}{\partial x}\left[u_\mathrm{n}\left(\frac{\gamma}{\gamma-1}p_\mathrm{n} + \frac{1}{2}\rho_\mathrm{n} u_\mathrm{n}^2 + \pi_\mathrm{n}\right)\right]$$
$$= \Lambda_\mathrm{n} + Q_{\mathrm{i}\to\mathrm{n}} + Q_{\mathrm{e}\to\mathrm{n}} + u_\mathrm{n} F_{\mathrm{c}\to\mathrm{n}} - \frac{1}{2}u_\mathrm{n}^2 M_\mathrm{n} \tag{5}$$

$$\frac{\partial}{\partial t}\left(\frac{1}{\gamma-1}p_\mathrm{i} + \frac{1}{2}\rho_\mathrm{i} u_\mathrm{c}^2 + \frac{B^2}{8\pi}\right)$$
$$+\frac{\partial}{\partial x}\left[u_\mathrm{c}\left(\frac{\gamma}{\gamma-1}p_\mathrm{i} + \frac{1}{2}\rho_\mathrm{i} u_\mathrm{c}^2 + \pi_\mathrm{i} + \frac{B^2}{4\pi}\right)\right]$$
$$= \Lambda_\mathrm{i} + Q_{\mathrm{n}\to\mathrm{i}} + Q_{\mathrm{e}\to\mathrm{i}} + u_\mathrm{c} F_{\mathrm{n}\to\mathrm{c}} - \frac{1}{2}u_\mathrm{c}^2 M_\mathrm{i} \tag{6}$$

$$\frac{\partial}{\partial t}\left(\frac{1}{\gamma-1}p_\mathrm{e} + \frac{1}{2}\rho_\mathrm{e} u_\mathrm{c}^2\right) + \frac{\partial}{\partial x}\left[u_\mathrm{c}\left(\frac{\gamma}{\gamma-1}p_\mathrm{e} + \frac{1}{2}\rho_\mathrm{e} u_\mathrm{c}^2\right)\right]$$
$$= \Lambda_\mathrm{e} + Q_{\mathrm{n}\to\mathrm{e}} + Q_{\mathrm{i}\to\mathrm{e}} - \frac{1}{2}u_\mathrm{c}^2 M_\mathrm{e} \tag{7}$$

$$\frac{\partial}{\partial t}(B) + \frac{\partial}{\partial x}(u_\mathrm{c} B) = 0 \tag{8}$$

Energy equation

Cooling

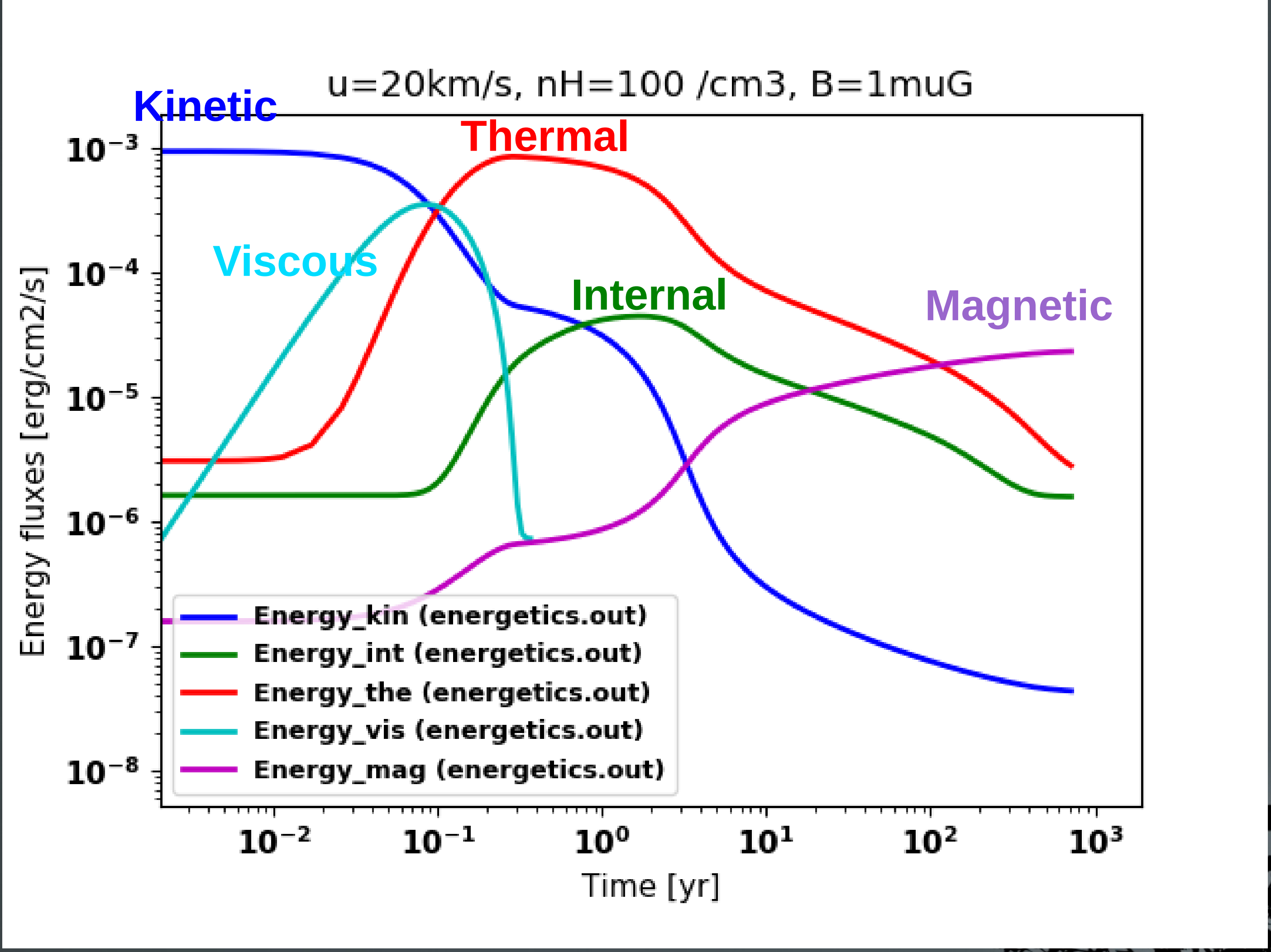
$$\partial_t \mathcal{E} + \partial_x \mathcal{F}_\mathcal{E} = -\Lambda$$

$$\mathcal{E} = \frac{P}{\gamma - 1} + \sum_{\text{excited level } i} E_i + \frac{1}{2}\rho u^2 + \frac{1}{8\pi} B^2$$

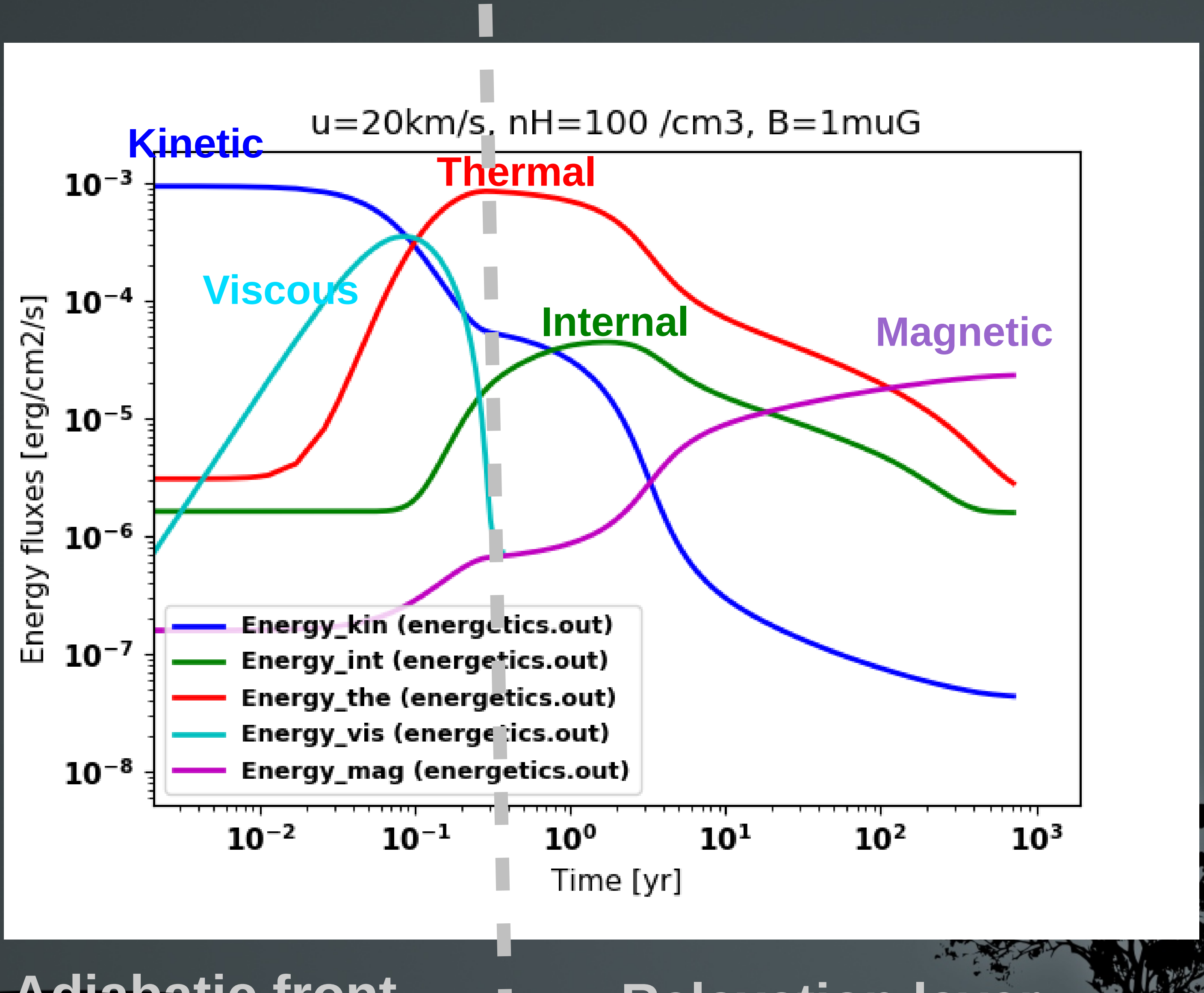
$$\mathcal{F}_\mathcal{E} = u \left(\frac{\gamma P}{\gamma - 1} + \sum_i E_i + \frac{1}{2}\rho u^2 + \frac{1}{4\pi} B^2 + \pi_x \right)$$

- Note: entrance energy flux often assumed ~ $\frac{1}{2} \rho u^3$
- Access total energy radiated by computing $[\mathcal{F}_\mathcal{E}]_{\text{pre}}^{\text{post}}$
- ISM is dilute: $\gamma = 5/3$ is a fairly good approximation

Energy fluxes through a steady shock in the ISM



Energy fluxes through a steady shock in the ISM



Adiabatic front

Relaxation layer

Shock profiles
examples and types

Adiabatic MHD

$$a = \frac{B}{\sqrt{4\pi\rho}}$$

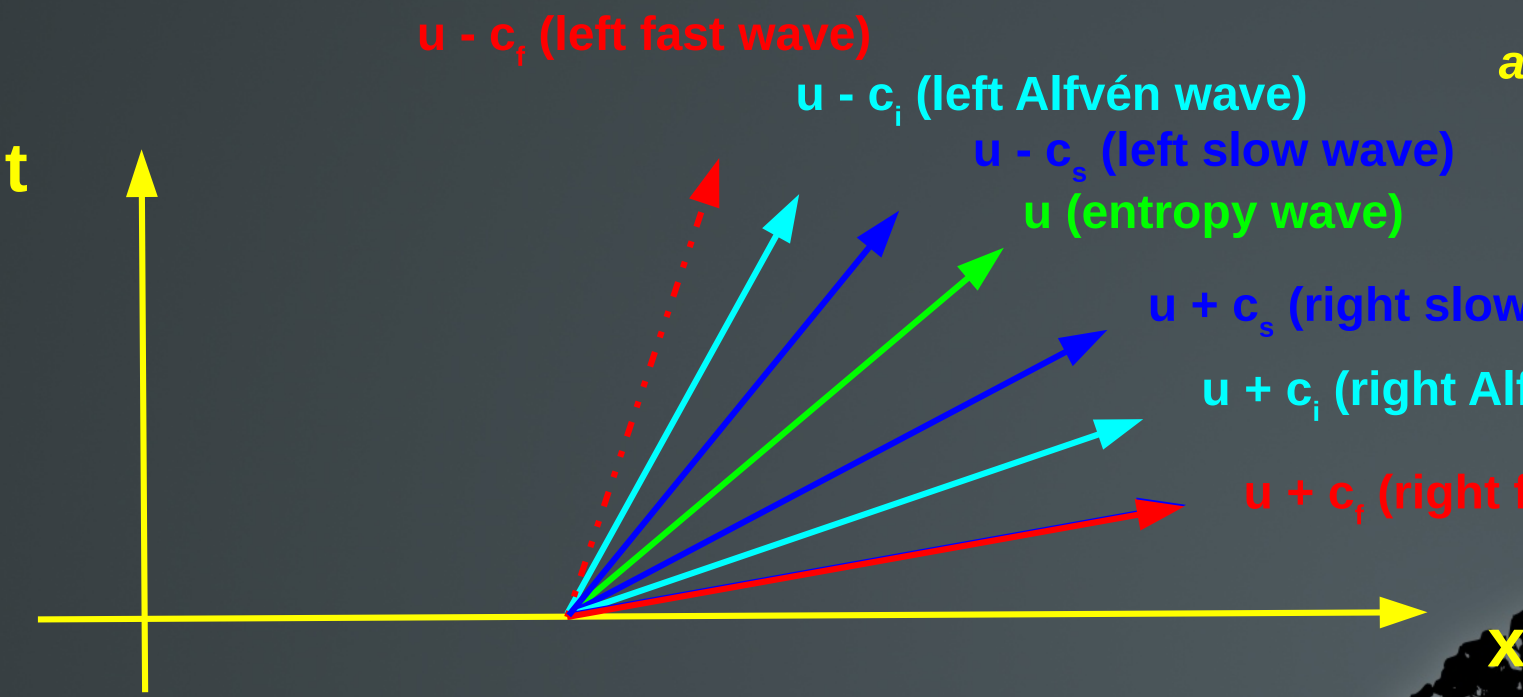
$$c_i = \frac{|B_x|}{\sqrt{4\pi\rho}}$$

$$c_s = \frac{1}{2}\sqrt{(c^2+a^2) - \sqrt{(c^2+a^2)^2 - 4c_i^2c^2}}$$

$$c_f = \frac{1}{2}\sqrt{(c^2+a^2) + \sqrt{(c^2+a^2)^2 - 4c_i^2c^2}}$$

$$c = \sqrt{\frac{\gamma P}{\rho}}$$

[plus equation on energy E and equation of state E(P,ρ)]



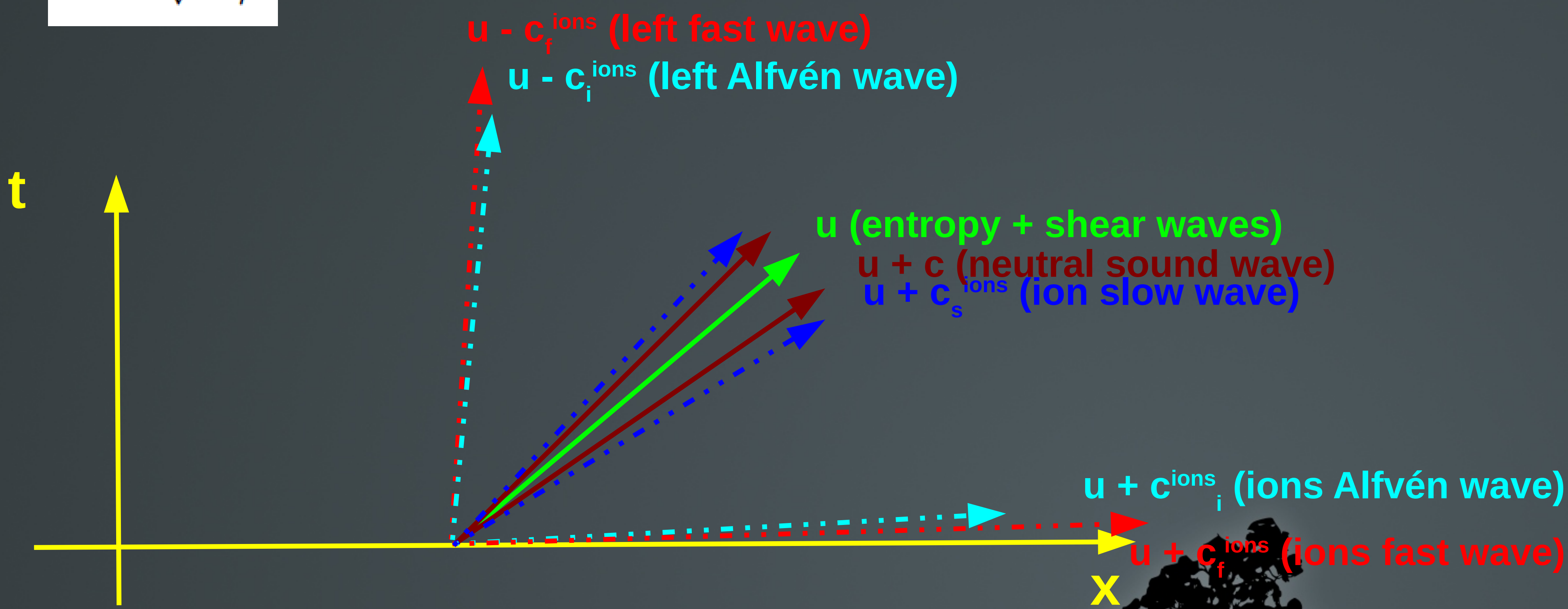
Two-fluids MHD

$$a = \frac{B}{\sqrt{4\pi\rho}}$$

$$c_i = \frac{|B_x|}{\sqrt{4\pi\rho}}$$

Alfvén wave in the charged fluid is much larger

For a transverse B field, c_s= c and c_i=c_f=a



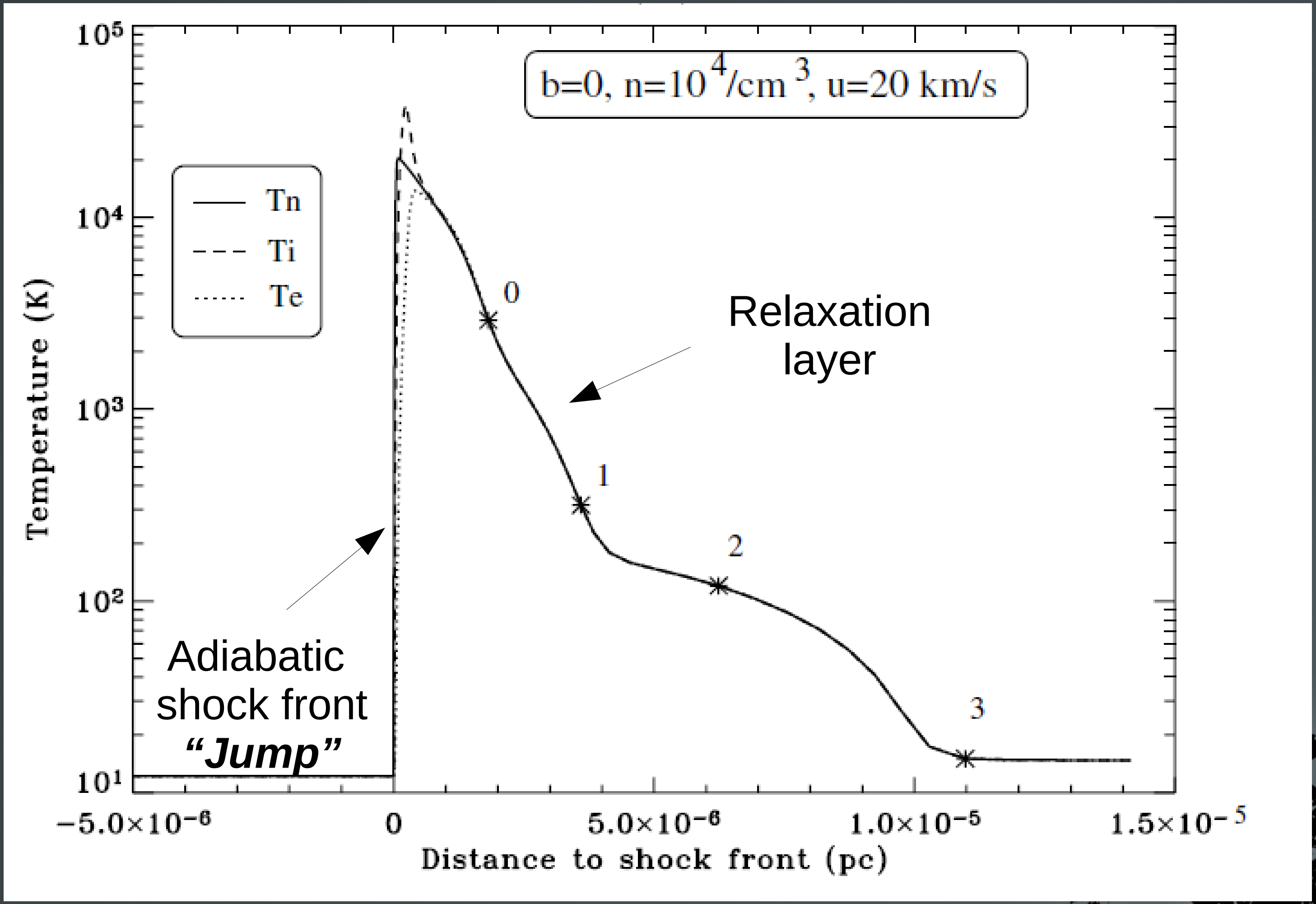
Unmagnetised (J-type) shock

(time-dependent simulation at steady state, Lesaffre+2004a)

$a < c_f^{ions} < u_0$

PRE

POST

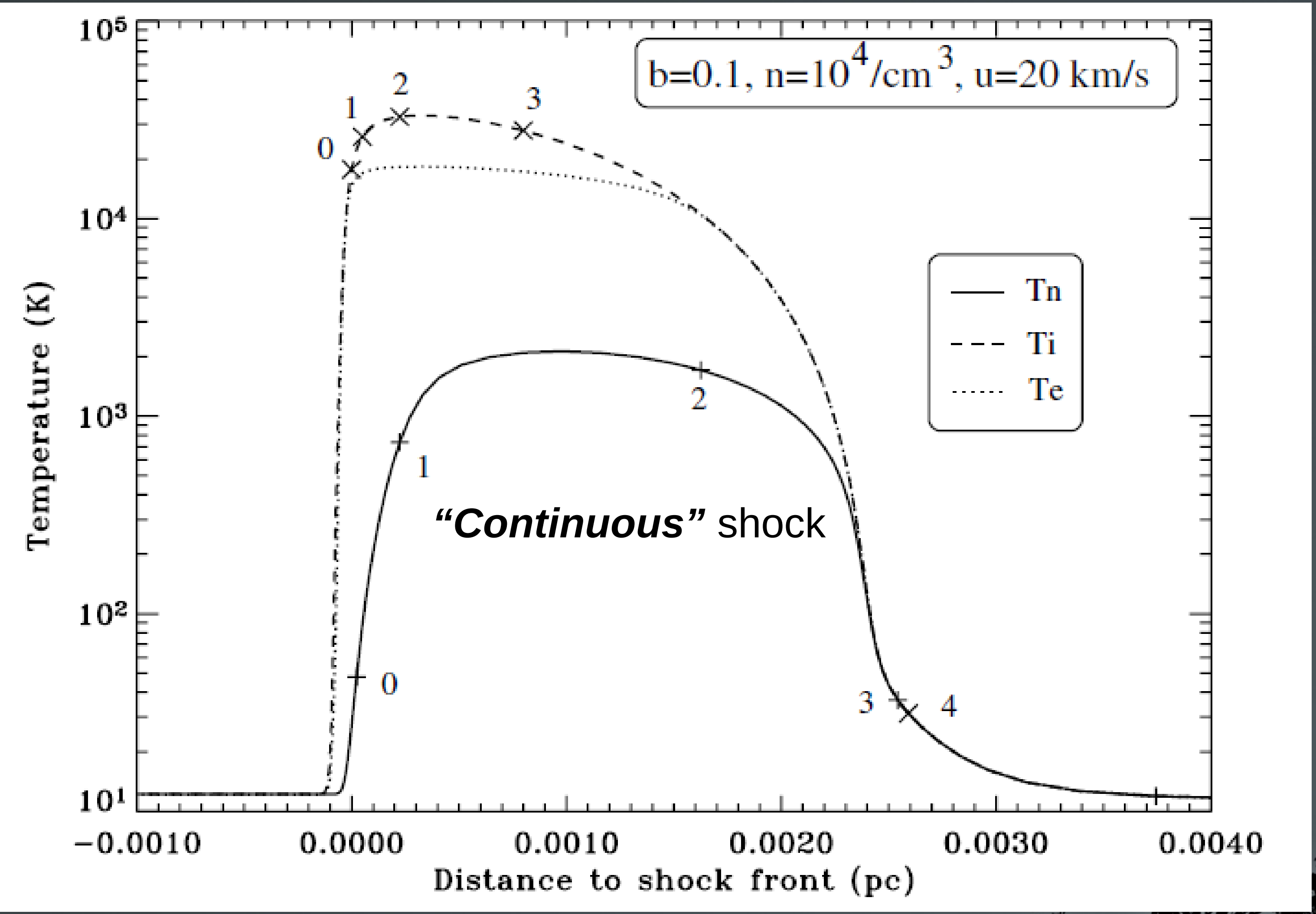


Strongly Magnetised (C-type) shock

(time-dependent simulation at steady state, Lesaffre+2004a)

$a < u_0 < c_f^{ions}$

PRE

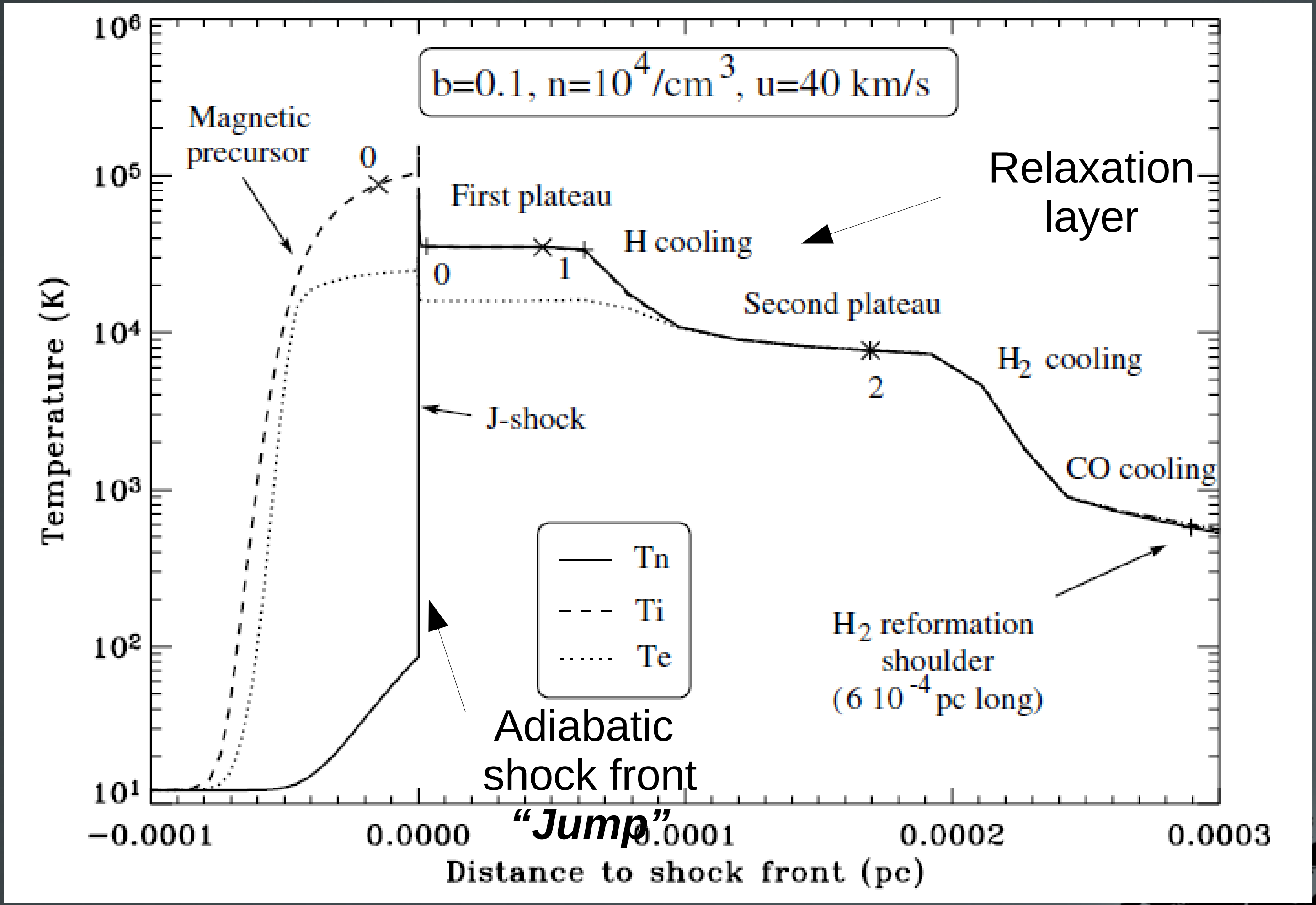


POST

Slightly Magnetised (JC-type) shock

(time-dependent simulation at steady state, Lesaffre+2004a)

PRE



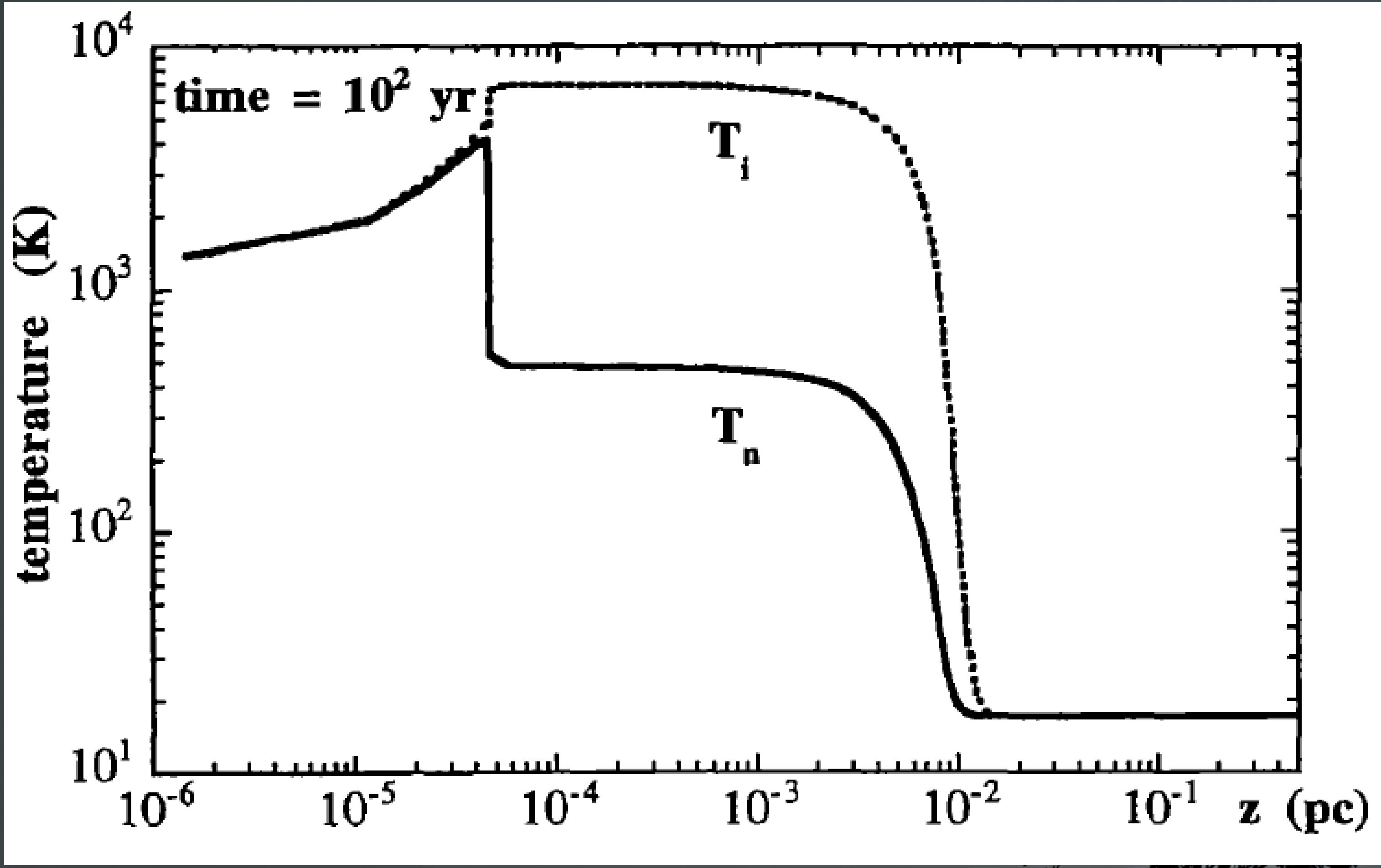
POST

A jump shock front is also present at early times of C-type shocks

Time-dependent models
ex: a C-type shock

Chièze, Pineau des Forêts, Flower (1998)

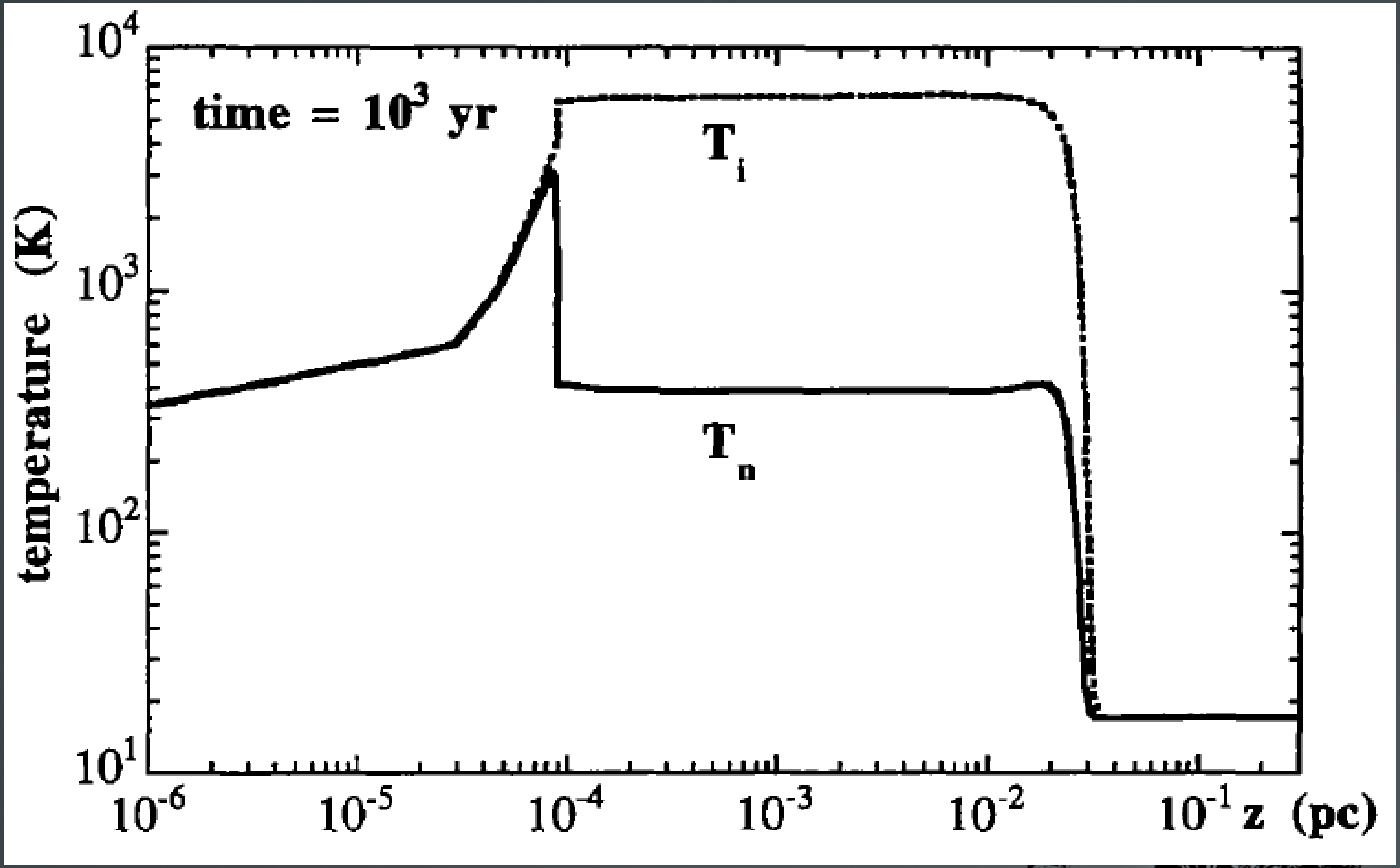
POST



PRE

Time-dependent models
ex: a C-type shock

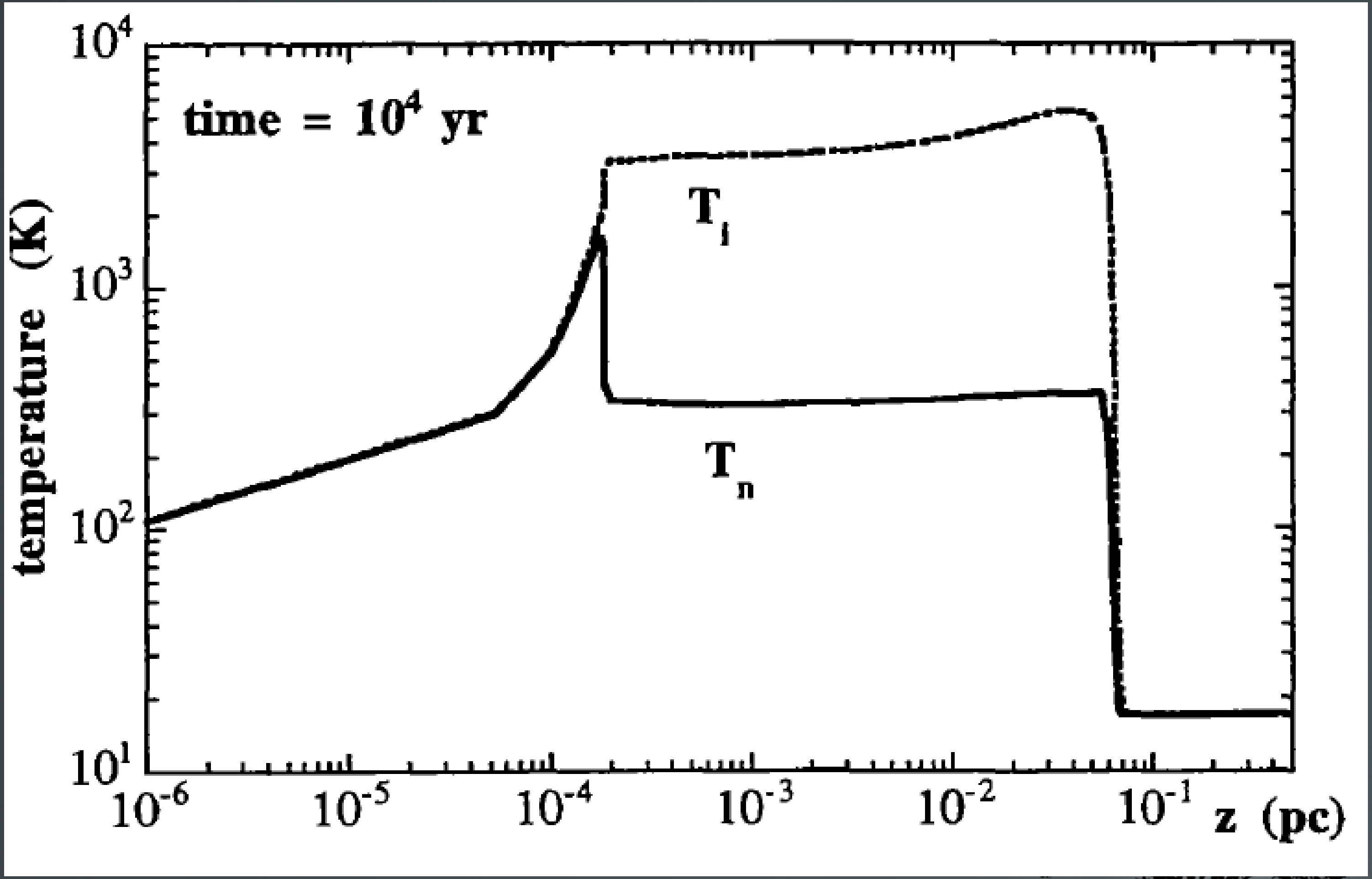
POST



PRE

Time-dependent models
ex: a C-type shock

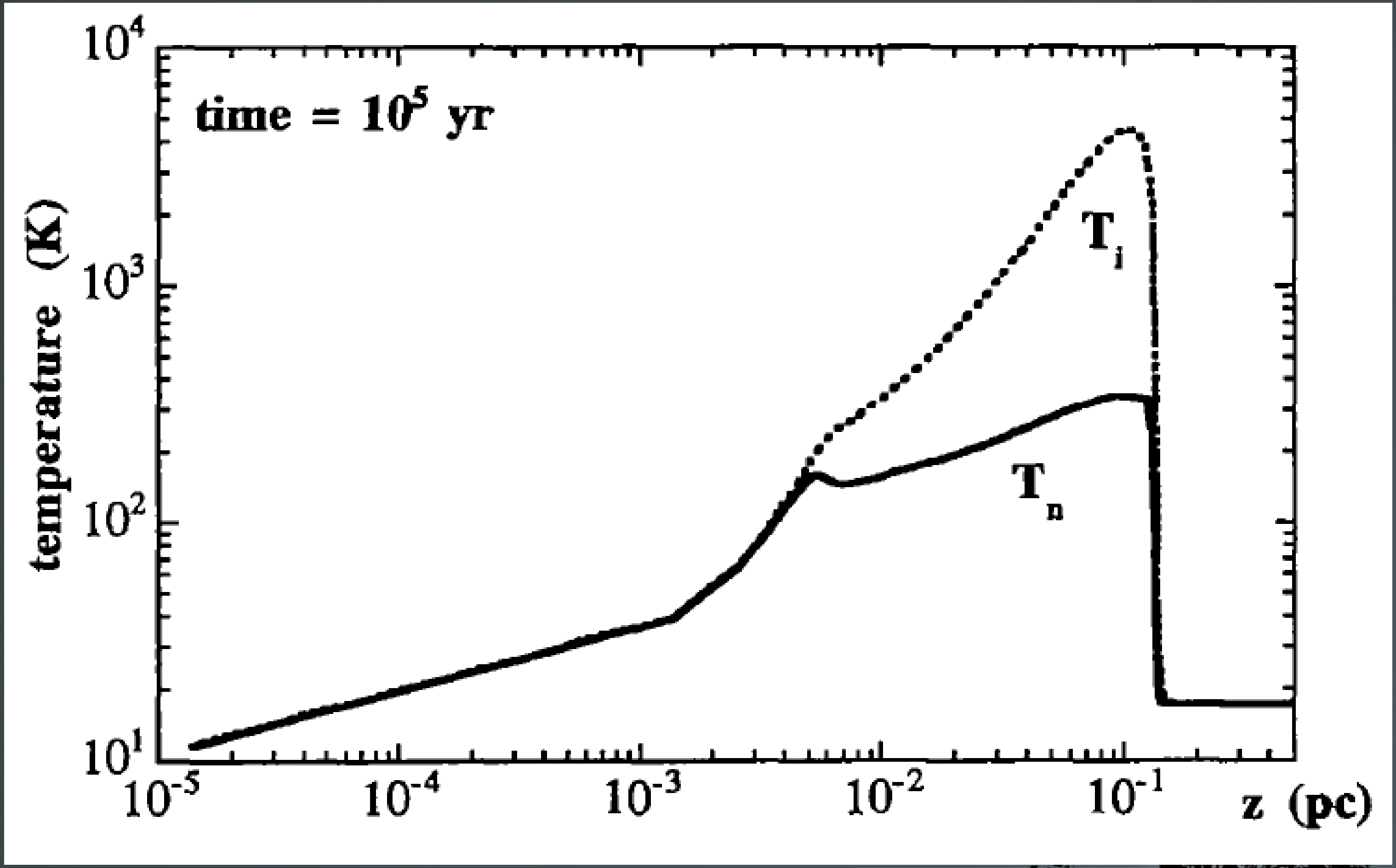
POST



PRE

Time-dependent models
ex: a C-type shock

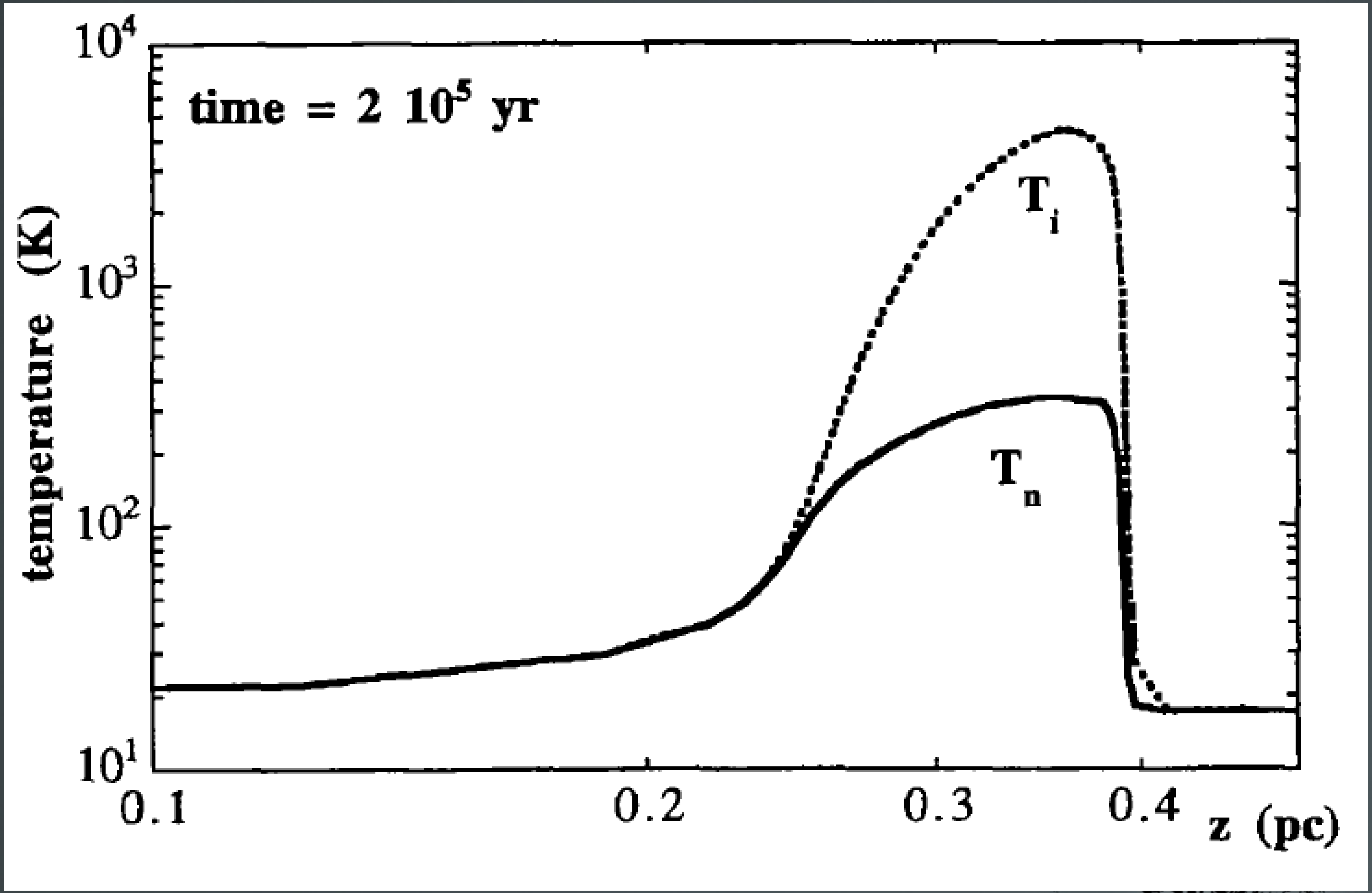
POST



PRE

Time-dependent models
<u>ex:</u> a C-type shock

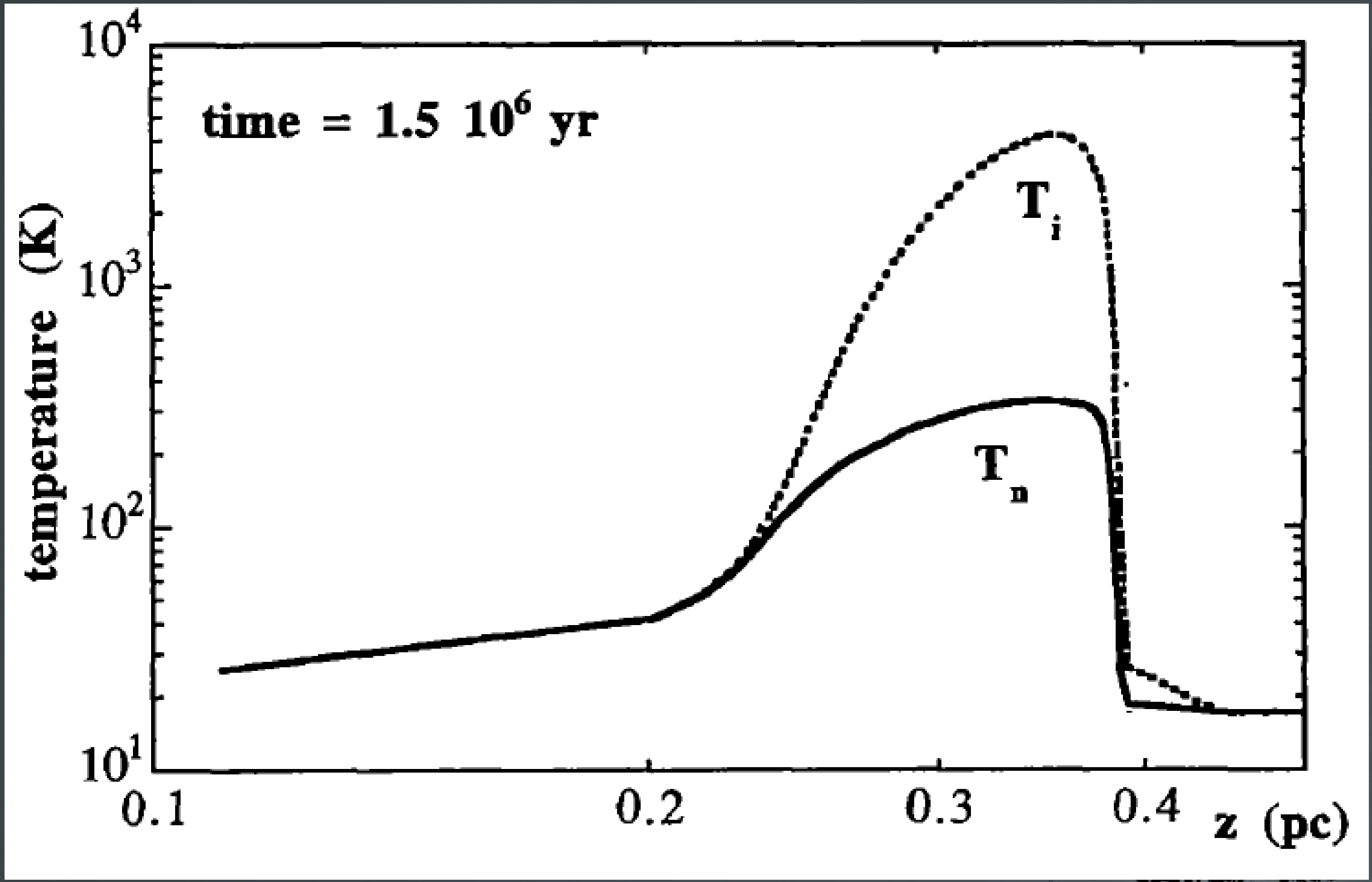
POST



PRE

Time-dependent models
ex: a C-type shock

POST

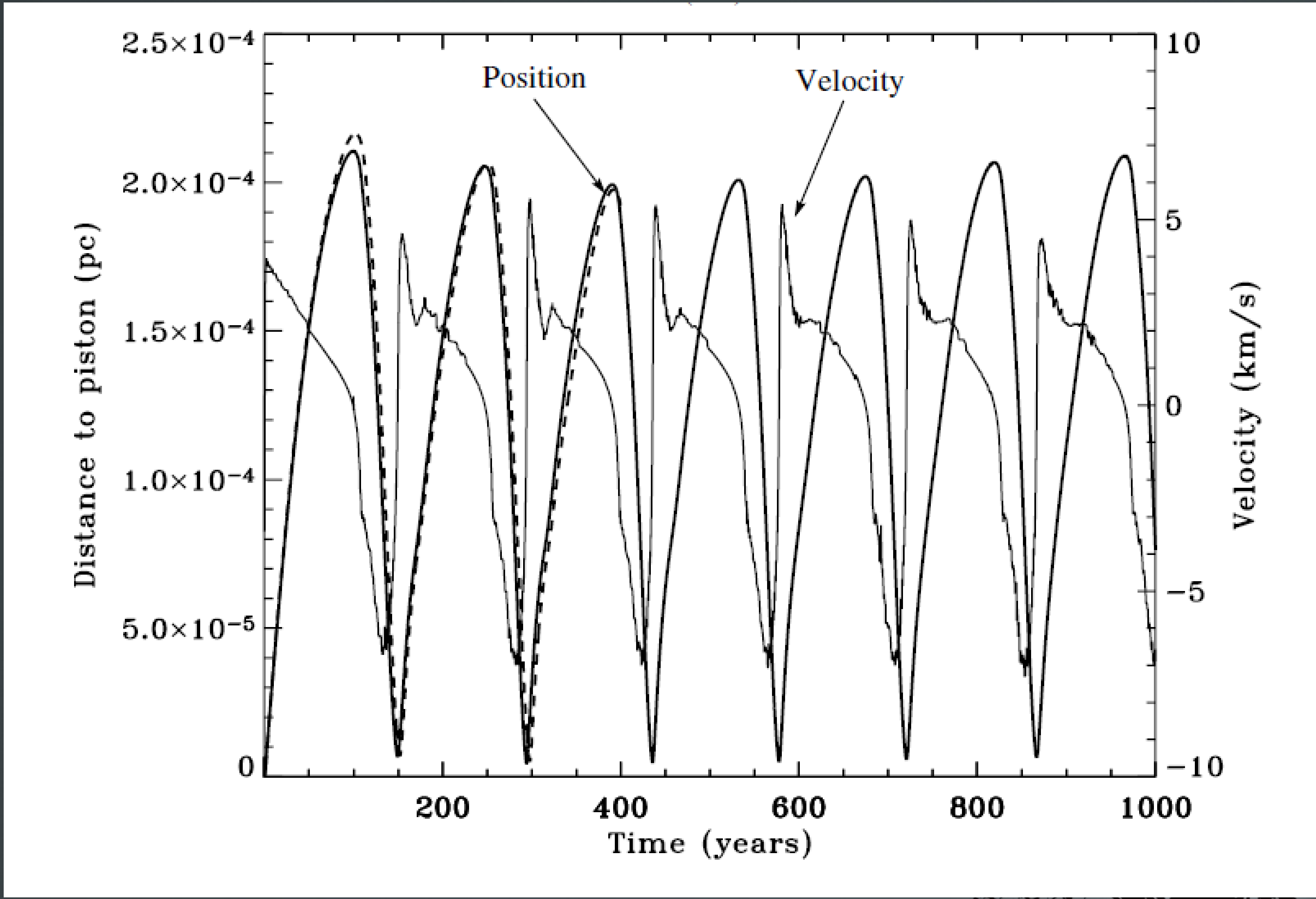


PRE

Shock stability

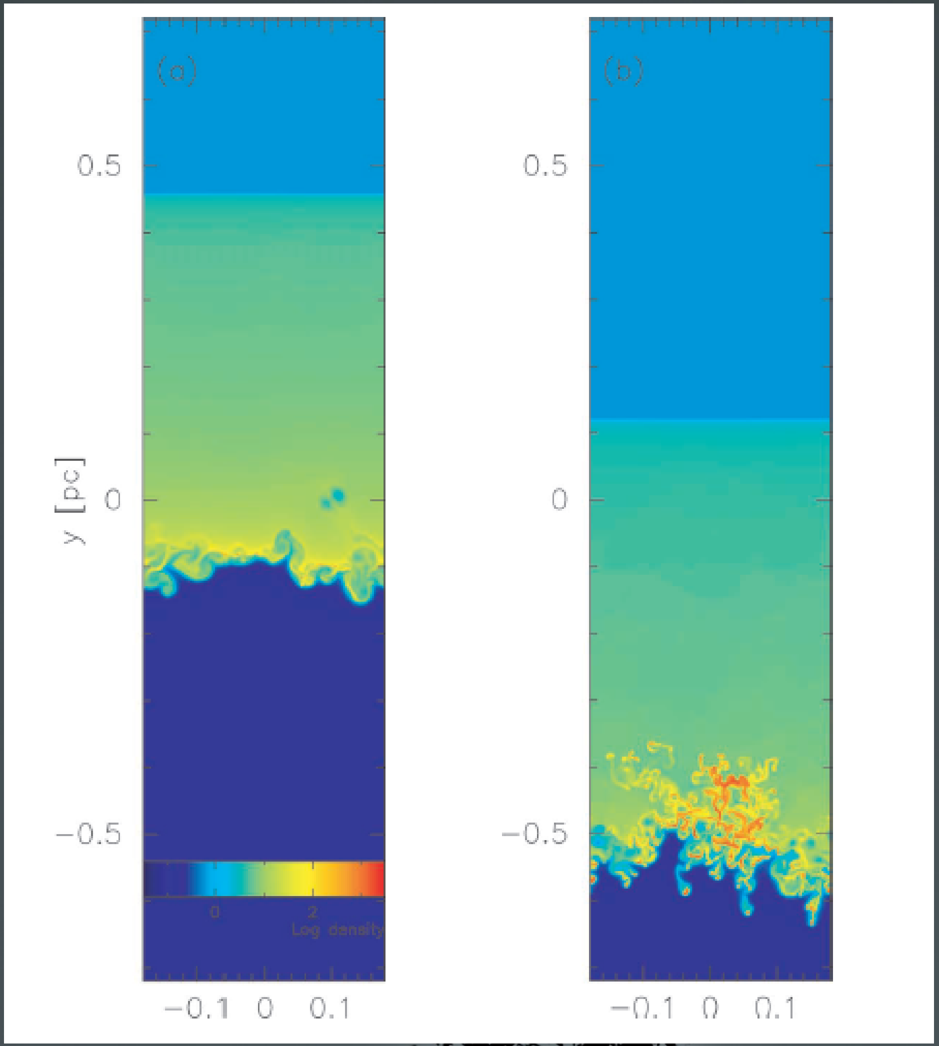
- Check Béthune (2023), coming out soon, for a generic method to test the linear stability of some discontinuities (some shocks, shear layers, contact discontinuities, ...).

Instability of shocks
Oscillatory instability (Lesaffre+2004a, Smith 2002, Chevalier&Imamura 1982)



Thermal instability in shocks (Koyama, Inutsuka 2002)

Some shocks can bring gas into a thermallly unstable state



Instability of C-type shocks
Wardle (1990) instability

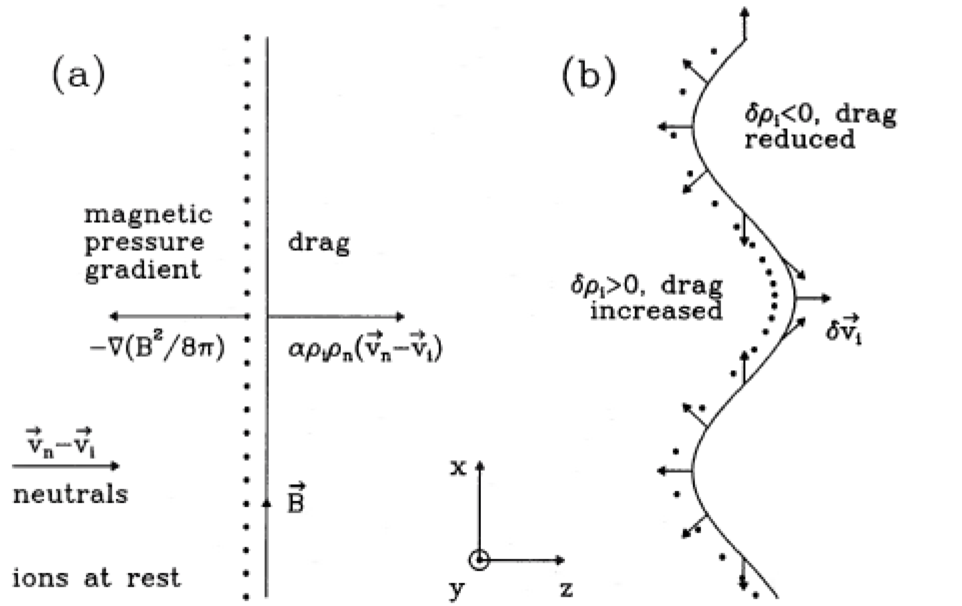
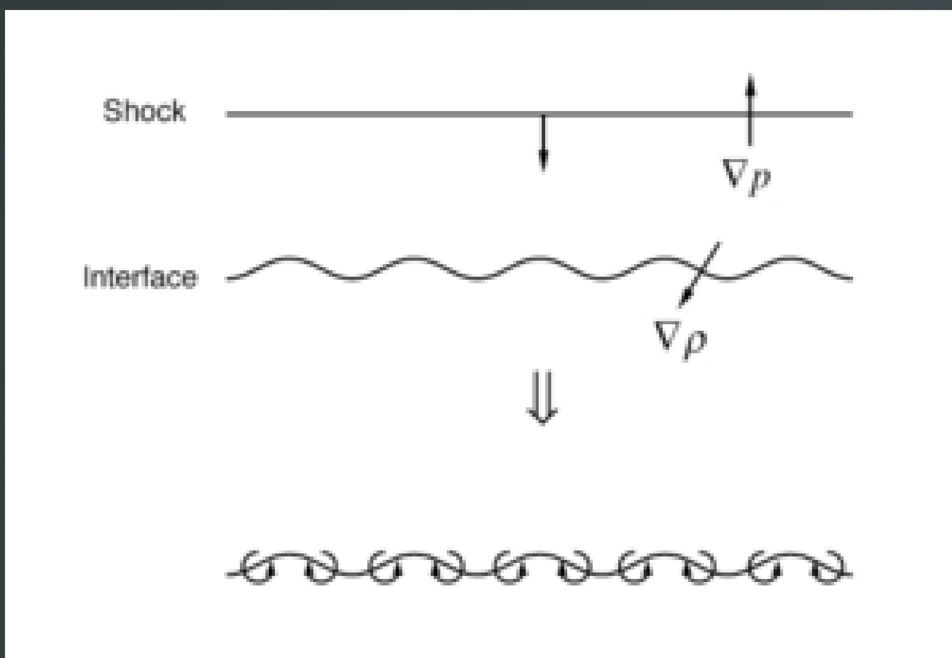


Figure 2. A possible mode of instability within the shock front, viewed from the frame instantaneously comoving with the ions. The ions are subjected to the magnetic-pressure gradient and the drag force caused by elastic collisions with the streaming neutral gas (a). If the field line is perturbed, (b), ions tend to collect in pockets and the drag force there overwhelms the pressure gradient. Meanwhile, the drag force is insufficient to constrain the field in the regions where the ion density is reduced. Both of these effects cause the perturbation to grow.

See Toth (1995) for simulations

Richtmyer Meshkov instability: a shock crosses a density interface



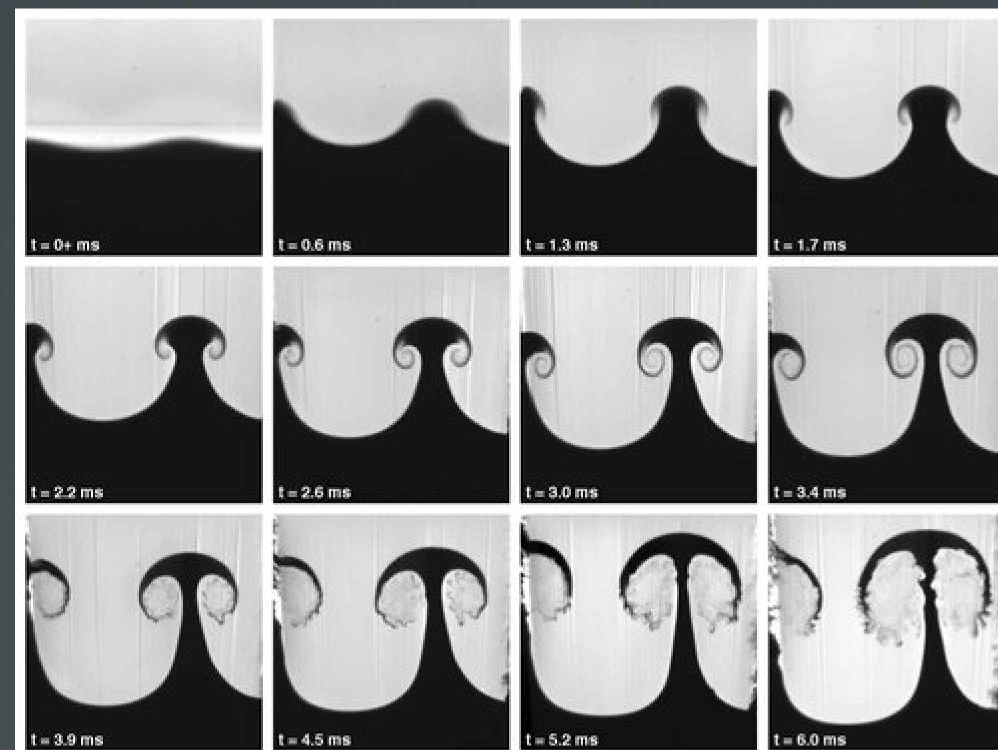


Figure 4: A sequence of PLIF images from the shock tube experiments described in Jacobs and Krivets (2005) in which a M=1.3 shock wave accelerates an air/SF6 interface.

Steady-state shocks and the Paris-Durham code

- Retrieve the code @ the URL :

  https://ism.obspm.fr/shock.html

Paris-Durham in a nutshell

- Born from long lived collaboration between David Flower in Durham and Guillaume Pineau des Forêts in Paris
- Paris-Durham solves all the conservation equations with DVODE solver (2-fluids, chemistry, molecular excitation: ~200 vars, few minutes)
- Many developpers have contributed, but main developper today is Benjamin Godard.
- Find it on the ISM services platform:

ism.obspm.fr/shocks.html

Other branches of Paris-Durham

- Dust grain physics and collisions (V. Guillet)
- Stellar Winds (L.N. Tram)
- Disc Winds (B. Tabone)
- Irradiated and self-irradiated shocks (B. Godard, A . Lehmann)
- Interface with DUMSES: CHEMSES

Time-dependent MHD shocks equations:

Complicated...

Two partial derivatives: time and space

Hard to solve:
prone to numerical instabilities and
large CPU cost (few hours for 32 chemical species)

$$\frac{\partial}{\partial t}(n_j) + \frac{\partial}{\partial x}(n_j u_\mathrm{n} + J_j) = R_j \quad \text{for } j \text{ neutral specie} \qquad (1)$$

$$\frac{\partial}{\partial t}(n_j) + \frac{\partial}{\partial x}(n_j u_\mathrm{c} + J_j) = R_j \quad \text{for } j \text{ ionic specie} \qquad (2)$$

$$\frac{\partial}{\partial t}(\rho_\mathrm{n} u_\mathrm{n}) + \frac{\partial}{\partial x}(\rho_\mathrm{n} u_\mathrm{n}^2 + p_\mathrm{n} + \pi_\mathrm{n}) = F_{\mathrm{c}\to\mathrm{n}} \qquad (3)$$

$$\frac{\partial}{\partial t}(\rho_\mathrm{c} u_\mathrm{c}) + \frac{\partial}{\partial x}\left(\rho_\mathrm{c} u_\mathrm{c}^2 + p_\mathrm{c} + \pi_\mathrm{i} + \frac{B^2}{8\pi}\right) = F_{\mathrm{n}\to\mathrm{c}} \qquad (4)$$

$$\frac{\partial}{\partial t}\left(\frac{1}{\gamma-1}p_\mathrm{n} + \frac{1}{2}\rho_\mathrm{n} u_\mathrm{n}^2\right) + \frac{\partial}{\partial x}\left[u_\mathrm{n}\left(\frac{\gamma}{\gamma-1}p_\mathrm{n} + \frac{1}{2}\rho_\mathrm{n} u_\mathrm{n}^2 + \pi_\mathrm{n}\right)\right]$$

$$= \Lambda_\mathrm{n} + Q_{\mathrm{i}\to\mathrm{n}} + Q_{\mathrm{e}\to\mathrm{n}} + u_\mathrm{n} F_{\mathrm{c}\to\mathrm{n}} - \frac{1}{2}u_\mathrm{n}^2 M_\mathrm{n} \qquad (5)$$

$$\frac{\partial}{\partial t}\left(\frac{1}{\gamma-1}p_\mathrm{i} + \frac{1}{2}\rho_\mathrm{i} u_\mathrm{c}^2 + \frac{B^2}{8\pi}\right)$$

$$+\frac{\partial}{\partial x}\left[u_\mathrm{c}\left(\frac{\gamma}{\gamma-1}p_\mathrm{i} + \frac{1}{2}\rho_\mathrm{i} u_\mathrm{c}^2 + \pi_\mathrm{i} + \frac{B^2}{4\pi}\right)\right]$$

$$= \Lambda_\mathrm{i} + Q_{\mathrm{n}\to\mathrm{i}} + Q_{\mathrm{e}\to\mathrm{i}} + u_\mathrm{c} F_{\mathrm{n}\to\mathrm{c}} - \frac{1}{2}u_\mathrm{c}^2 M_\mathrm{i} \qquad (6)$$

$$\frac{\partial}{\partial t}\left(\frac{1}{\gamma-1}p_\mathrm{e} + \frac{1}{2}\rho_\mathrm{e} u_\mathrm{c}^2\right) + \frac{\partial}{\partial x}\left[u_\mathrm{c}\left(\frac{\gamma}{\gamma-1}p_\mathrm{e} + \frac{1}{2}\rho_\mathrm{e} u_\mathrm{c}^2\right)\right]$$

$$= \Lambda_\mathrm{e} + Q_{\mathrm{n}\to\mathrm{e}} + Q_{\mathrm{i}\to\mathrm{e}} - \frac{1}{2}u_\mathrm{c}^2 M_\mathrm{e} \qquad (7)$$

$$\frac{\partial}{\partial t}(B) + \frac{\partial}{\partial x}(u_\mathrm{c} B) = 0 \qquad (8)$$

Steady-state MHD shocks equations:

Still complicated...

ONE partial derivative: time and space

Still hard to solve:
prone to other numerical instabilities
but on the shelf methods exist
and VERY SMALL CPU cost (0.1 s for 32 species)

$$\cancel{\frac{\partial}{\partial t}(n_j)} + \frac{\partial}{\partial x}(n_j u_\mathrm{n} + J_j) = R_j \quad \text{for } j \text{ neutral specie} \tag{1}$$

$$\cancel{\frac{\partial}{\partial t}(n_j)} + \frac{\partial}{\partial x}(n_j u_\mathrm{c} + J_j) = R_j \quad \text{for } j \text{ ionic specie} \tag{2}$$

$$\cancel{\frac{\partial}{\partial t}(\rho_\mathrm{n} u_\mathrm{n})} + \frac{\partial}{\partial x}(\rho_\mathrm{n} u_\mathrm{n}^2 + p_\mathrm{n} + \pi_\mathrm{n}) = F_{\mathrm{c}\to\mathrm{n}} \tag{3}$$

$$\cancel{\frac{\partial}{\partial t}(\rho_\mathrm{c} u_\mathrm{c})} + \frac{\partial}{\partial x}\left(\rho_\mathrm{c} u_\mathrm{c}^2 + p_\mathrm{c} + \pi_\mathrm{i} + \frac{B^2}{8\pi}\right) = F_{\mathrm{n}\to\mathrm{c}} \tag{4}$$

$$\frac{\partial}{\partial t}\left(\cancel{\frac{1}{\gamma-1}p_\mathrm{n}} + \cancel{\frac{1}{2}\rho_\mathrm{n} u_\mathrm{n}^2}\right) + \frac{\partial}{\partial x}\left[u_\mathrm{n}\left(\frac{\gamma}{\gamma-1}p_\mathrm{n} + \frac{1}{2}\rho_\mathrm{n} u_\mathrm{n}^2 + \pi_\mathrm{n}\right)\right]$$
$$= \Lambda_\mathrm{n} + Q_{\mathrm{i}\to\mathrm{n}} + Q_{\mathrm{e}\to\mathrm{n}} + u_\mathrm{n} F_{\mathrm{c}\to\mathrm{n}} - \frac{1}{2}u_\mathrm{n}^2 M_\mathrm{n} \tag{5}$$

$$\frac{\partial}{\partial t}\left(\cancel{\frac{1}{\gamma-1}p_\mathrm{i}} + \cancel{\frac{1}{2}\rho_\mathrm{i} u_\mathrm{c}^2} + \cancel{\frac{B^2}{8\pi}}\right)$$
$$+ \frac{\partial}{\partial x}\left[u_\mathrm{c}\left(\frac{\gamma}{\gamma-1}p_\mathrm{i} + \frac{1}{2}\rho_\mathrm{i} u_\mathrm{c}^2 + \pi_\mathrm{i} + \frac{B^2}{4\pi}\right)\right]$$
$$= \Lambda_\mathrm{i} + Q_{\mathrm{n}\to\mathrm{i}} + Q_{\mathrm{e}\to\mathrm{i}} + u_\mathrm{c} F_{\mathrm{n}\to\mathrm{c}} - \frac{1}{2}u_\mathrm{c}^2 M_\mathrm{i} \tag{6}$$

$$\frac{\partial}{\partial t}\left(\cancel{\frac{1}{\gamma-1}p_\mathrm{e}} + \cancel{\frac{1}{2}\rho_\mathrm{e} u_\mathrm{c}^2}\right) + \frac{\partial}{\partial x}\left[u_\mathrm{c}\left(\frac{\gamma}{\gamma-1}p_\mathrm{e} + \frac{1}{2}\rho_\mathrm{e} u_\mathrm{c}^2\right)\right]$$
$$= \Lambda_\mathrm{e} + Q_{\mathrm{n}\to\mathrm{e}} + Q_{\mathrm{i}\to\mathrm{e}} - \frac{1}{2}u_\mathrm{c}^2 M_\mathrm{e} \tag{7}$$

$$\cancel{\frac{\partial}{\partial t}(B)} + \frac{\partial}{\partial x}(u_\mathrm{c} B) = 0 \tag{8}$$

Numerical methods for ODE

- Implicit schemes with Newton-Raphson
- DVODE
- MEBDFI
- Exponential integrators

Newton-Raphson

- d$\mathbf{y}$/dt = $\mathbf{f}$ ($\mathbf{y}$,t)
- Implicit schemes use Newton-Raphson to solve:

$$\mathbf{y}_{new}-\mathbf{y}_{old} = \Delta t \;.\; \mathbf{f}\,(\,\mathbf{y}_{old}+\alpha.(\mathbf{y}_{new}-\mathbf{y}_{old})\,,\,t)$$

- Unconditionnaly stable (α>0.5) but needs N-R to converge... not always easy !
- Semi-implicit (linear approximation ($\mathbf{f} \rightarrow$ d$\mathbf{f}$/d$\mathbf{y}$.) for α=1) fast but inaccurate
- DVODE is a higher order version of this method
- MEBDFI is similar, but handles algebraic equations, too

Other branches of Paris-Durham

- Dust grain physics and collisions (V. Guillet)
- Stellar Winds (L.N. Tram)
- Disc Winds (B. Tabone)
- Irradiated and self-irradiated shocks (B. Godard, A . Lehmann)
- Interface with DUMSES: CHEMSES

Heating/Cooling processes

- Ion-neutral & Viscous friction
- Atomic line cooling: C+, C, O, S, N, Si, Fe, O+, S+, N+, Si+, Fe+, Lyman α
- Molecular cooling (H2, H_2O, CO, OH, & isotopes)
- Cosmic ray ionisation heating
- Photo-electric heating on grains
- Grains thermal collisional coupling
- Chemical reactions heating

Chemistry

- Versatile networks (mainly 32/150 or 130/1300 species/reactions)
- 2 & 3-body reactions (charge exchange, dissociation, radiative recombination, endo/exo thermic reactions, deuteration..)
- Photo-reactions (dissociation, ionisation)
- Cosmic ray induced reactions (direct, and secondary)
- Grain surface catalysed reactions (essentially H2,HD formation)
- Adsorption, desorption (drift+thermal), mantle sputtering, core erosion, photo-desorption

Line emission

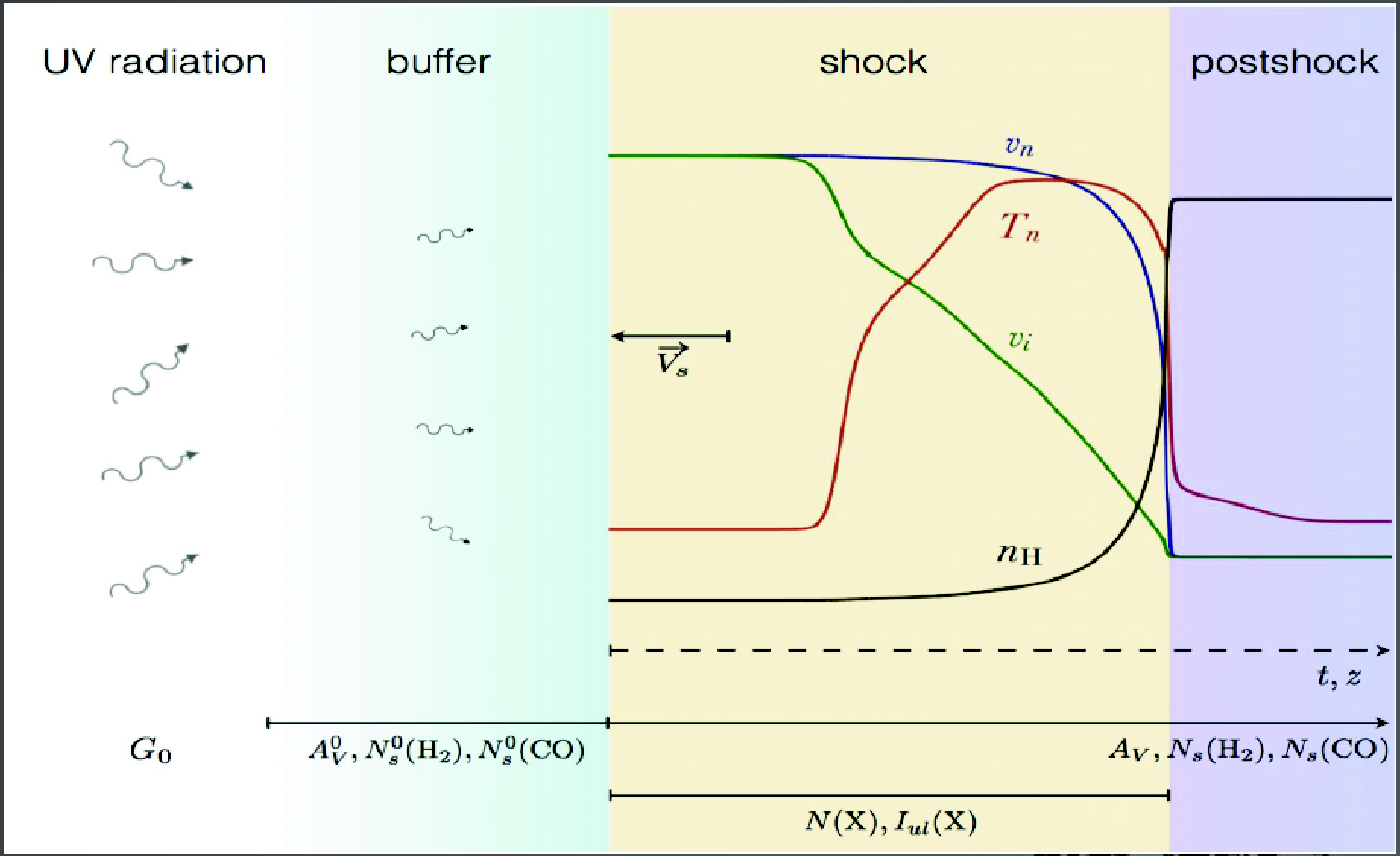
- Time-dependent excitation of H_2 => H_2 excitation diagrams directly ready in output
- Post-processed LVG radiative transfer in several molecular ladders (CO, SiO, H_2O) [Gusdorf, Godard]

Essential to interpret observations.

- On the fly transfer (Flower & Pineau des forêts, CO, SiO, H_2O, OH, and methanol)
- Optically thin line shape modelling (Tram+2018)

Irradiated shocks
shocks ↔ PDR

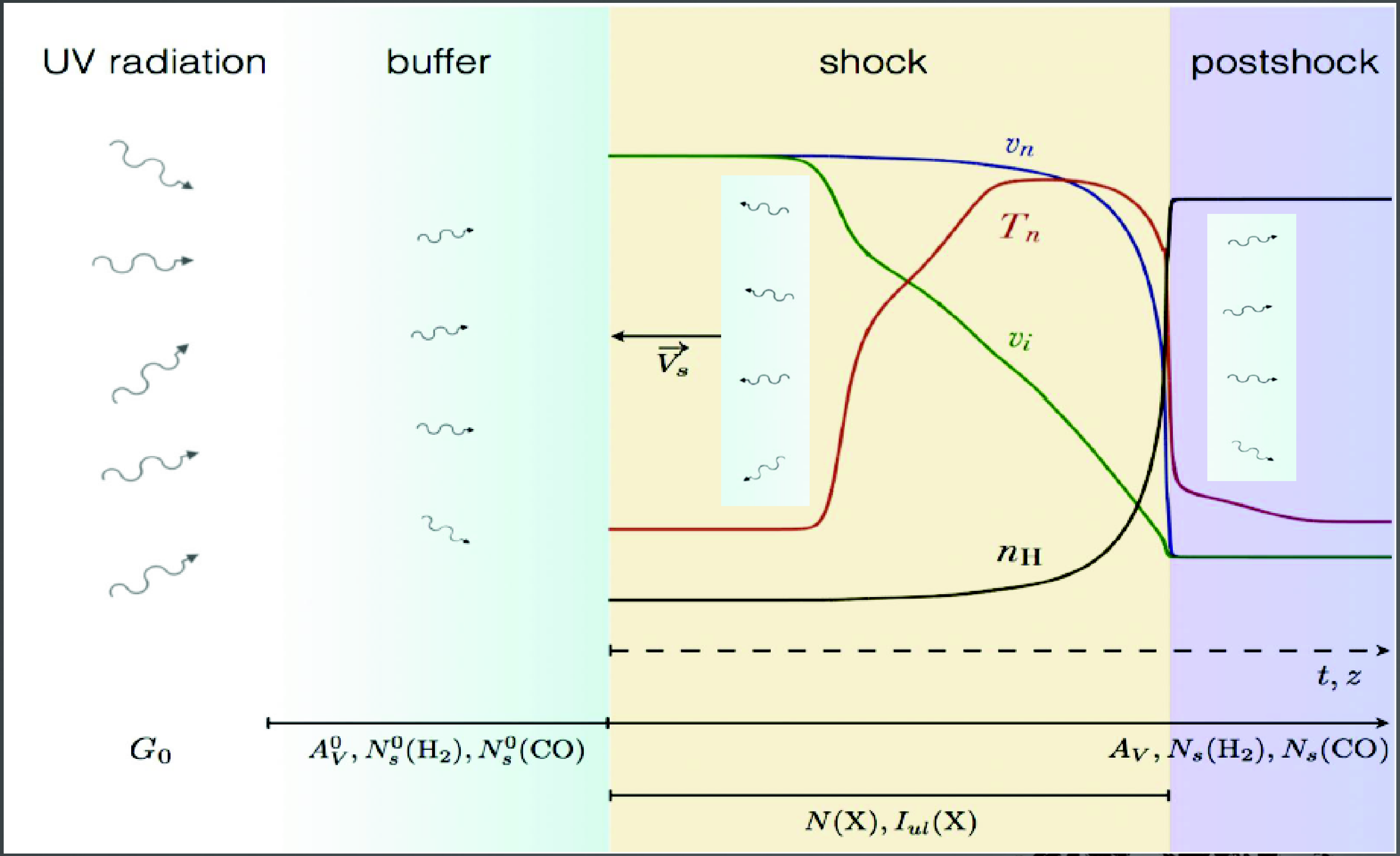
Credits: B.Godard



Godard+2019

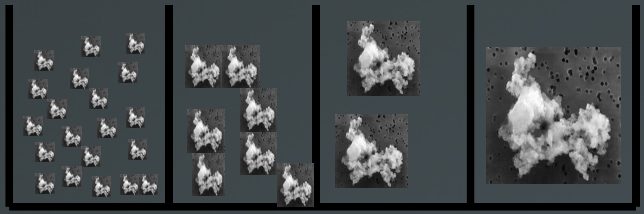
Self-Irradiated shocks

Credits: B.Godard



Lehman+2020,2022

Grain physics in magnetised shocks



- Shattering and coagulation: bin by bin population of sizes, charges (+,-,0) (V.Guillet 2010)
  - Even neutral grains are coupled to magnetic fields
  - Small grains influence ion-neutral coupling : strong interplay shattering ↔ coupling ↔ thermal structures
- Rotation and disruption (Thiem Hoang & Tram 2018, submitted)

Shocks in more than 1D

Quasi-steady shocks in 3D (Richard et al. 2022)

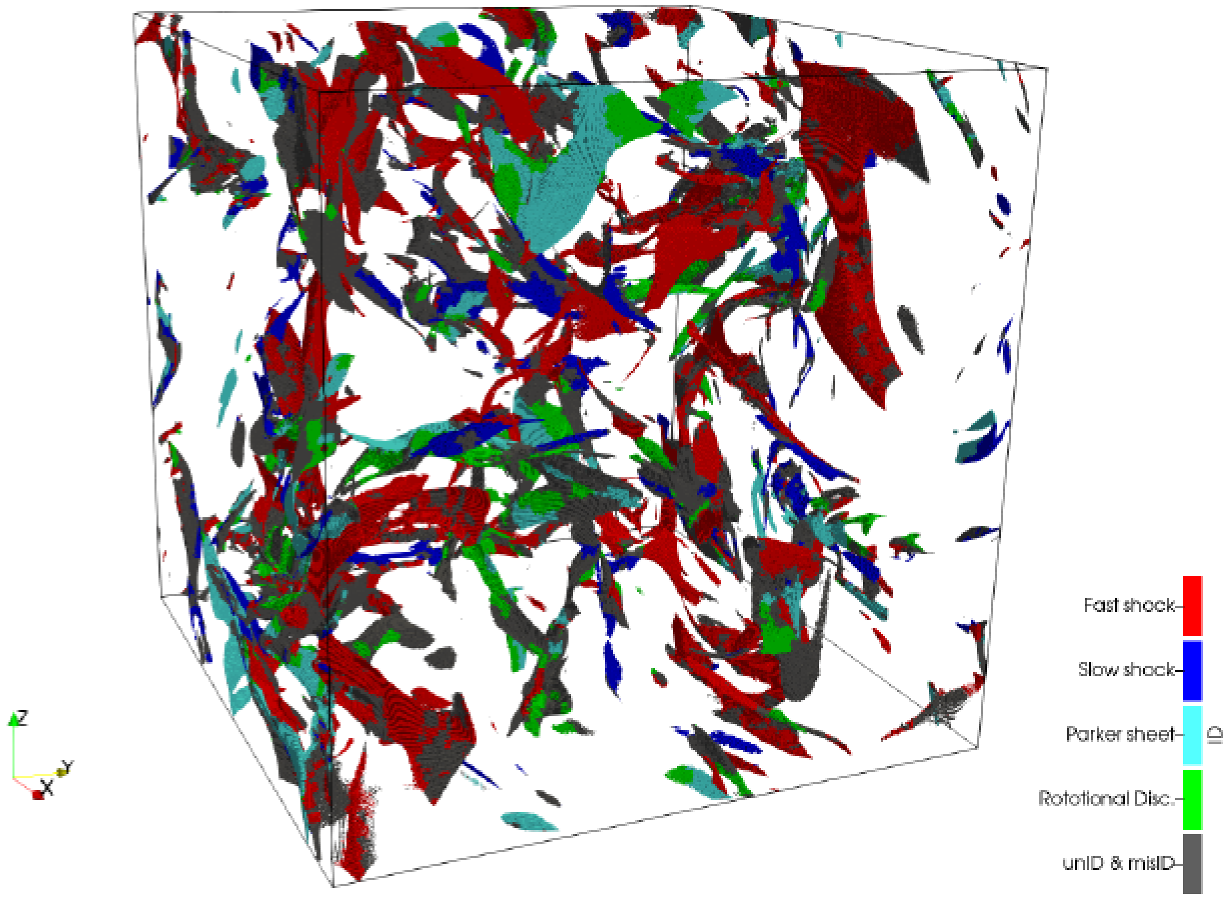
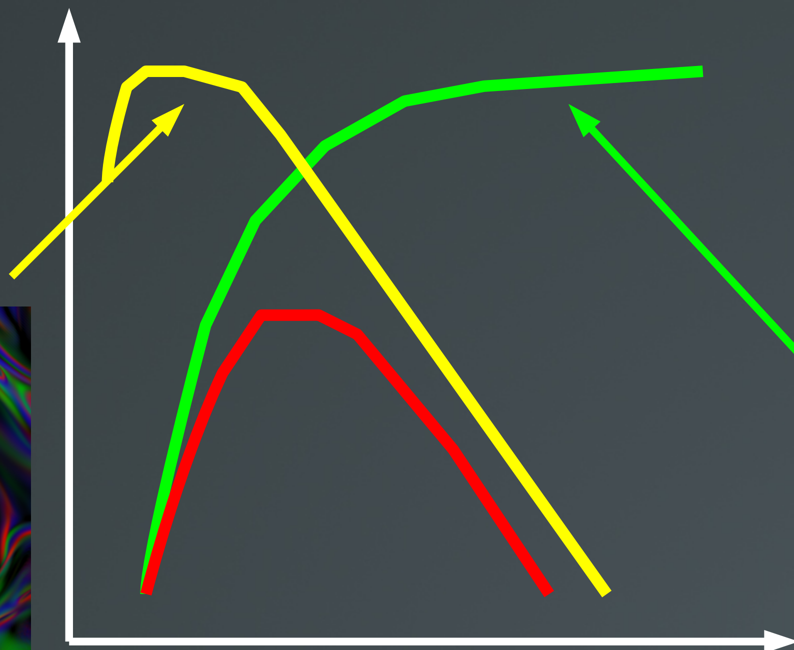


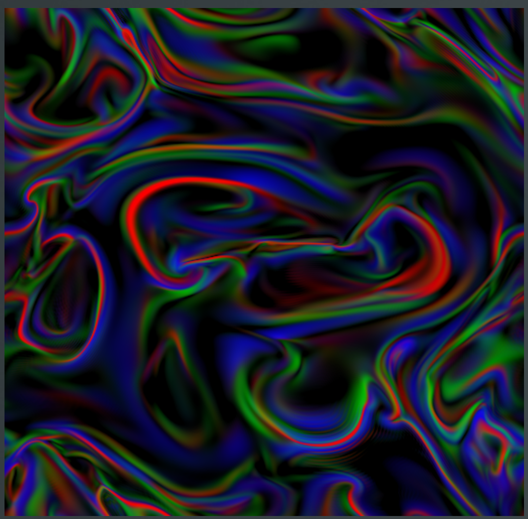
FIG. V.9 *Structures à forte dissipation extraites d'une simulation aux conditions initiales OT à* $\mathcal{P}_\mathrm{m} = 1$. *Le pas de temps de cette sortie est* $t \simeq 1/3 t_\mathrm{turnover}$.

Prospects

Intermittent statistics of the dissipation

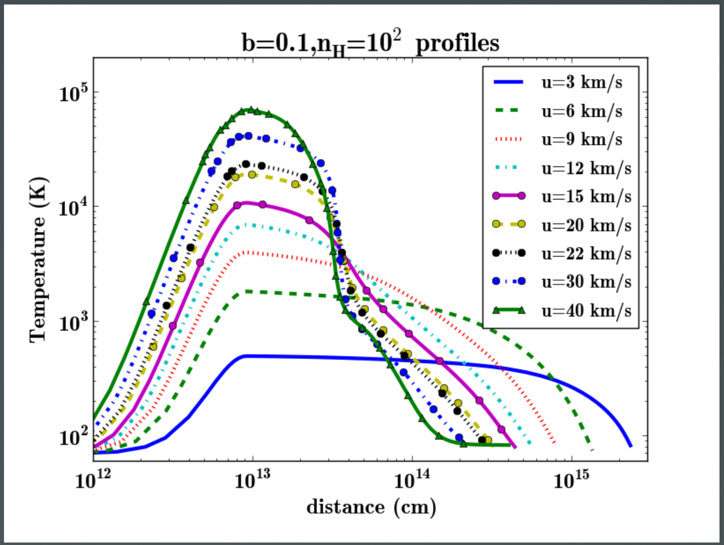


Molecular yields from Shocks (for example)

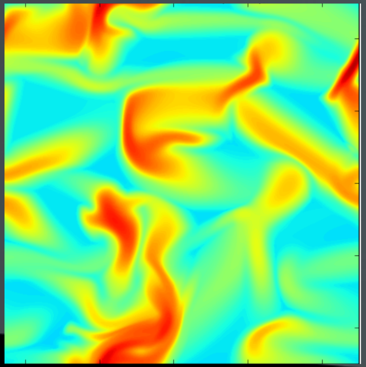


3D simulations
Richard et al. (2022)
(also Momferratos et al. 2013)

=> Molecules
Formation + excitation

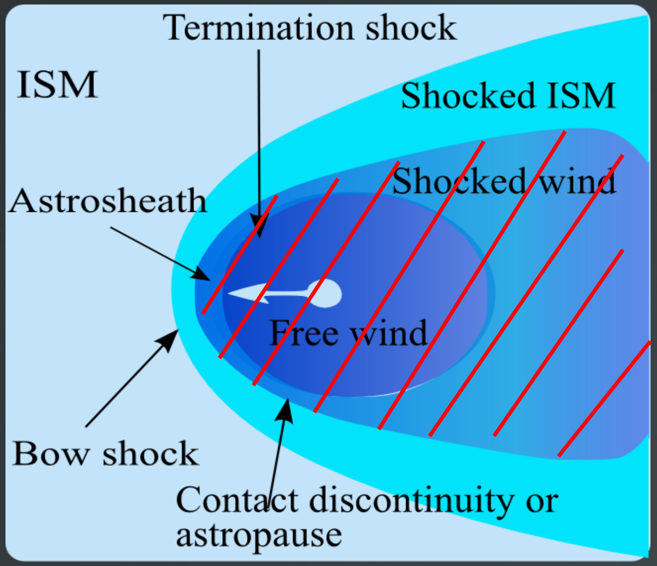


1D simulations



CO map (Lesaffre + 2020)

Validation with 2D simulations

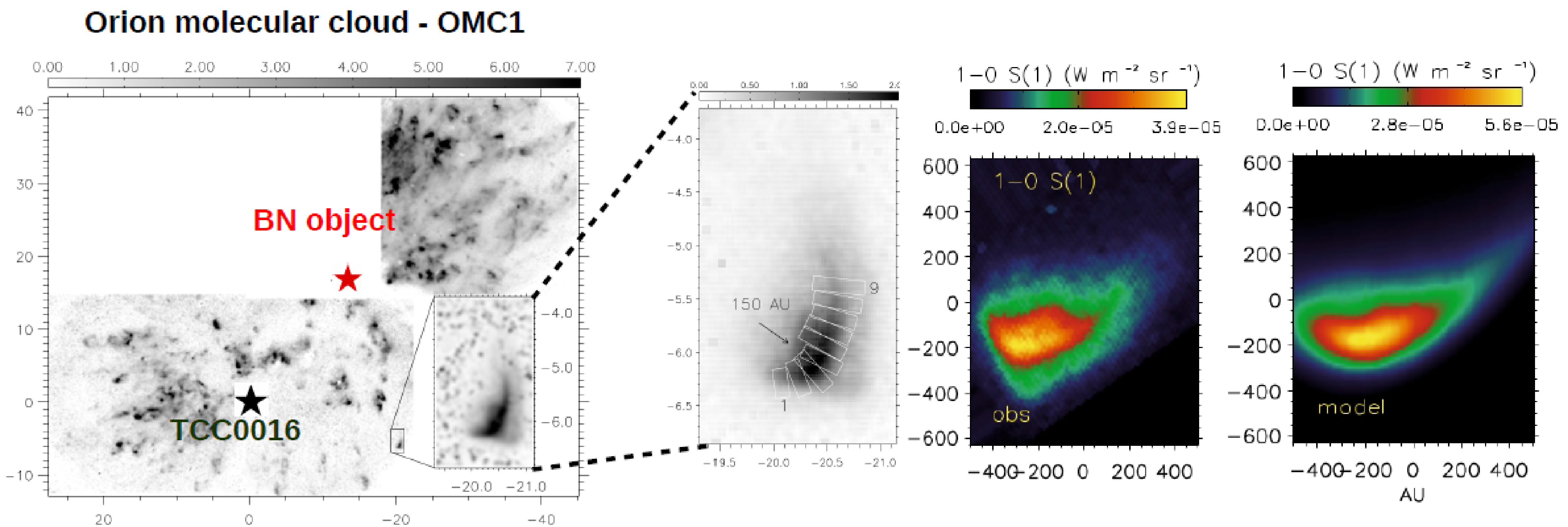


Modeling 3D bow shocks
Previous work

H_2 emission models

Slice models — Map models

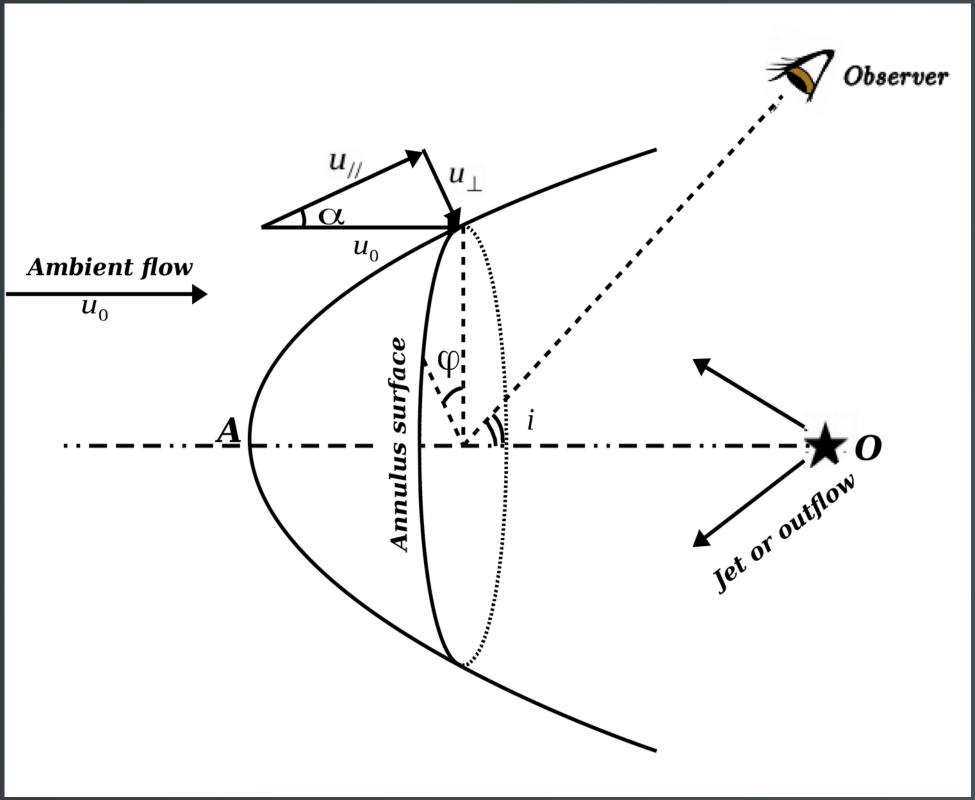
Kristensen et al. (2007) — Gustafsson et al. (2010)



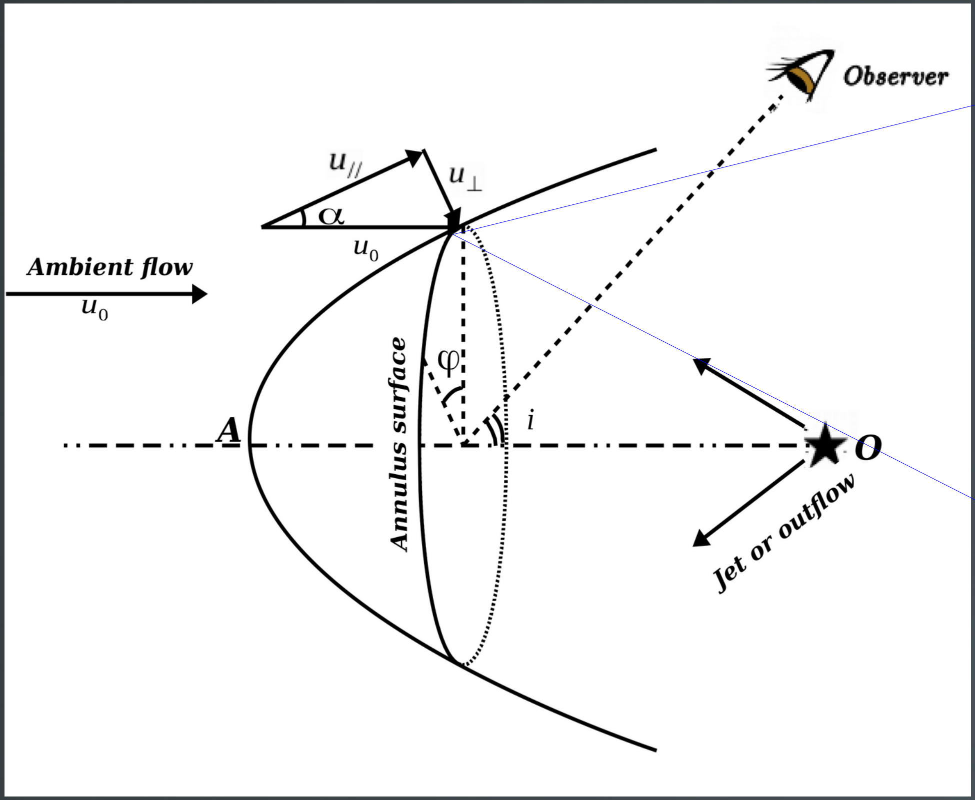
We assume the 3D bow shock is a collection of 1D planar shocks

<u>This amounts to neglect:</u>
sideways gradients and friction, curvature radius, geometrical dilution

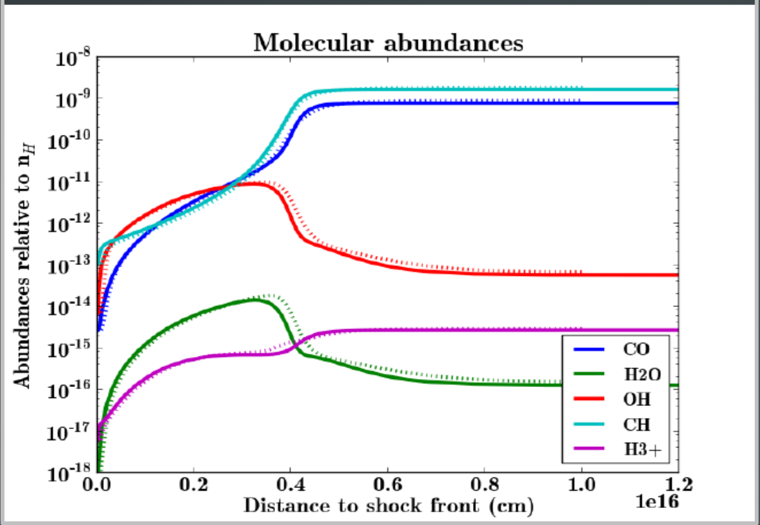
<u>New :</u> shock age, arbitrary shape, excitation diagrams and line profiles, G_0>0



We assume the 3D bow shock is a collection of 1D planar shocks



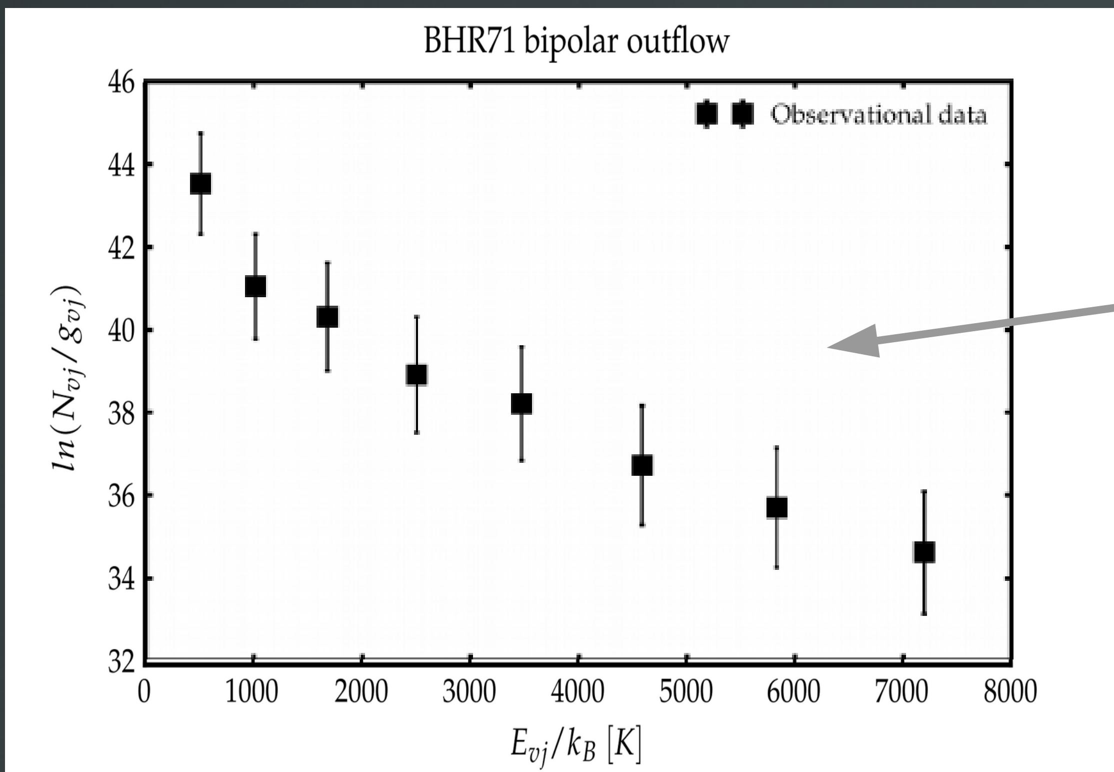
steady-state shock at 3 km/s



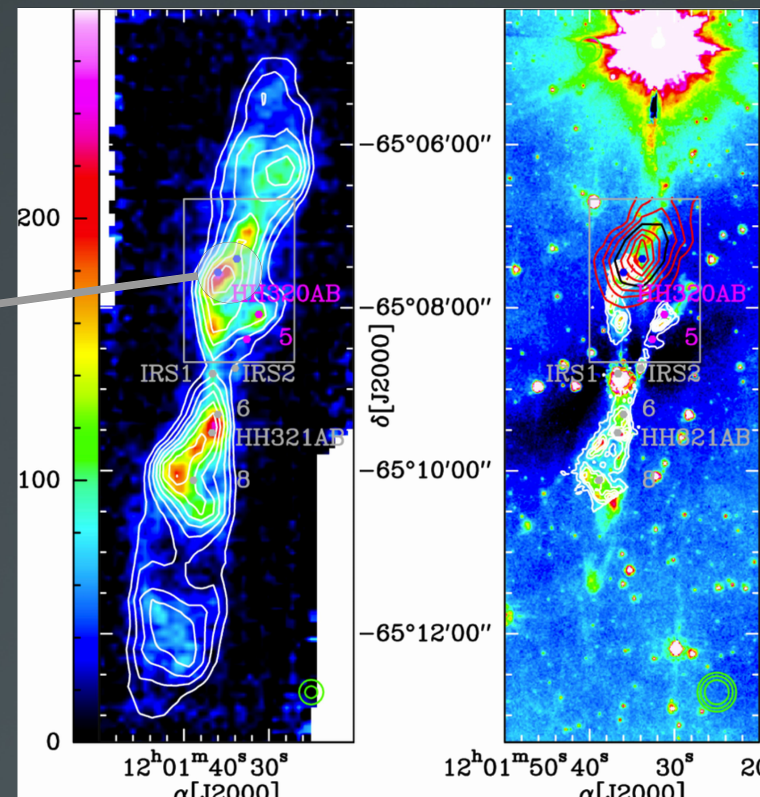
Models with the Paris-Durham[1] Shock code

Application to a protostellar jet
H_2 emission in BHR71

Tram et al. (2018)



Gusdorf et al. (2015)

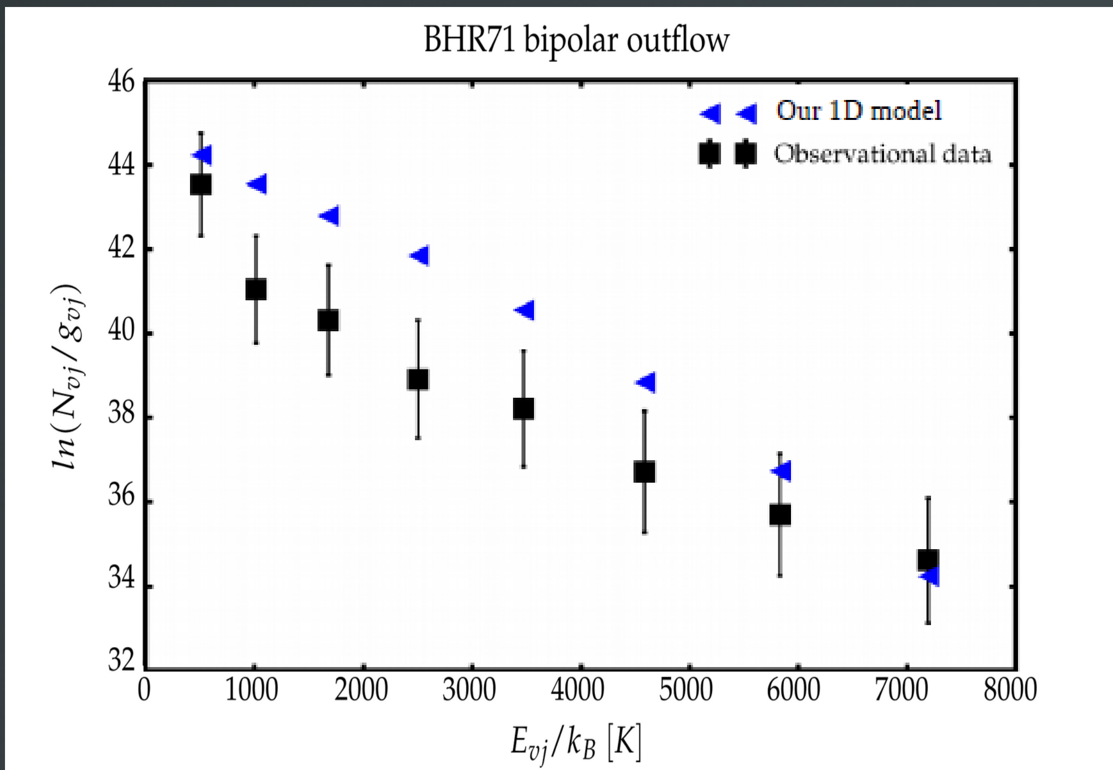


Map: CO(6-5)
Contours:
CO(3-2)

Map: 8 micron
Contours:
H_2 0-0S(5)
SiO(5-4)

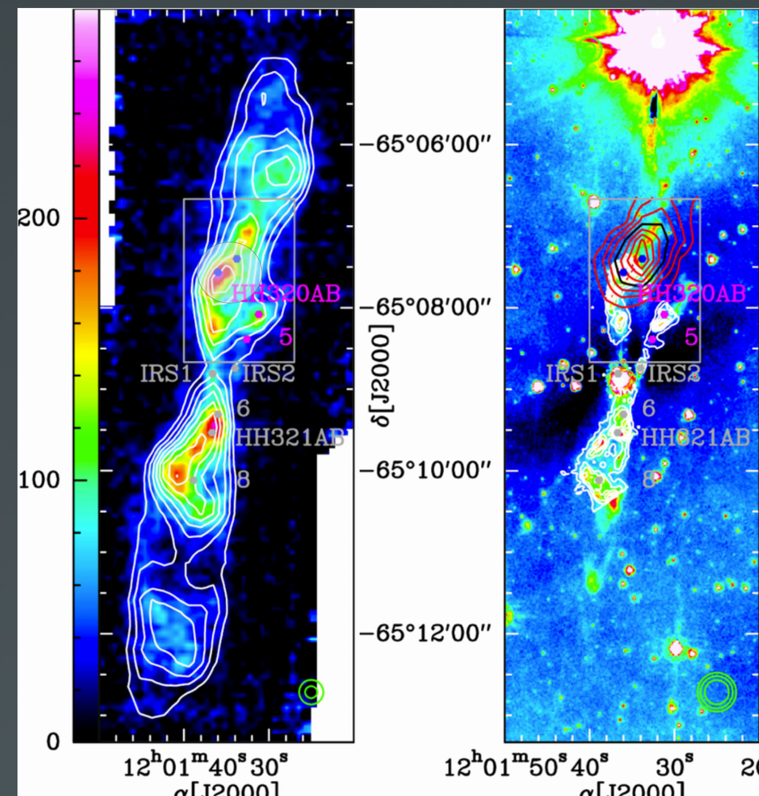
H_2 emission in BHR71

Tram et al. (2018)



Parameter	Value
n_H	10^4 cm^{-3}
Age	10^3 yr
$\Delta u_\perp$	21–23 km s^{-1}
b_0	1.5

Gusdorf et al. (2015)



Map: CO(6-5)
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H_2 emission in BHR71

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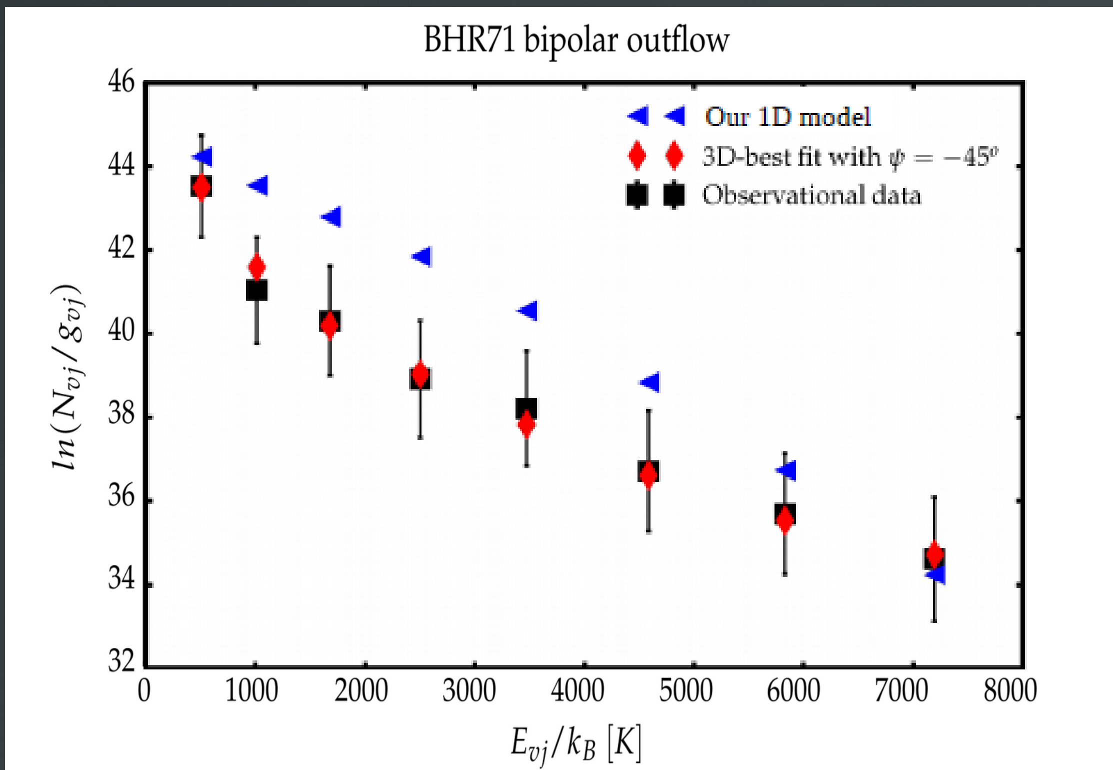
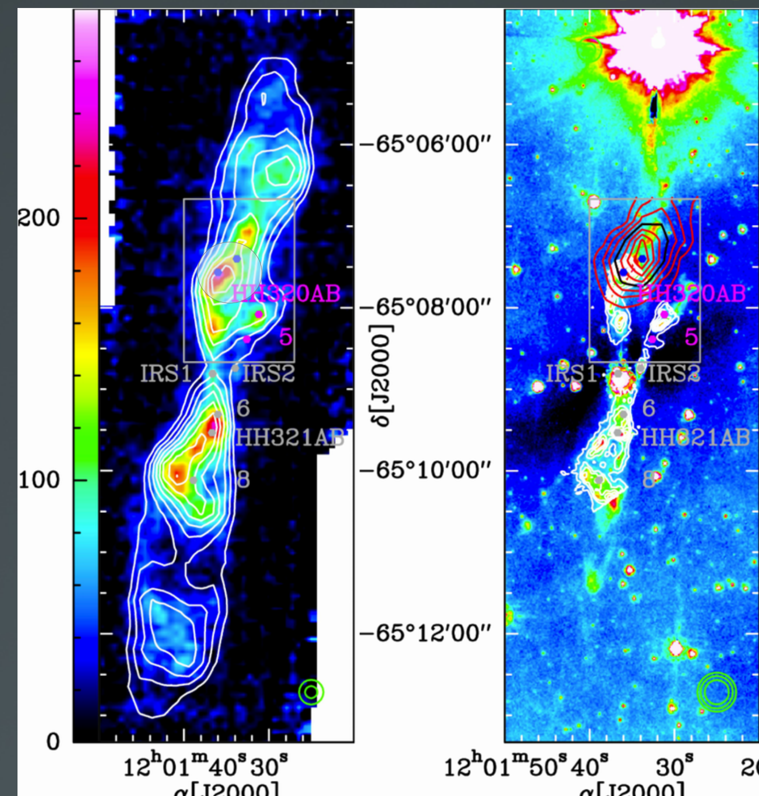


Parameter	Value
n_H	10^4 cm^{-3}
Age	10^3 yr
$\Delta u_\perp$	21–23 km s^{-1}
b_0	1.5
ψ	$-50^o \pm 20^o$
u_0 and β	NA

Gusdorf et al. (2015)

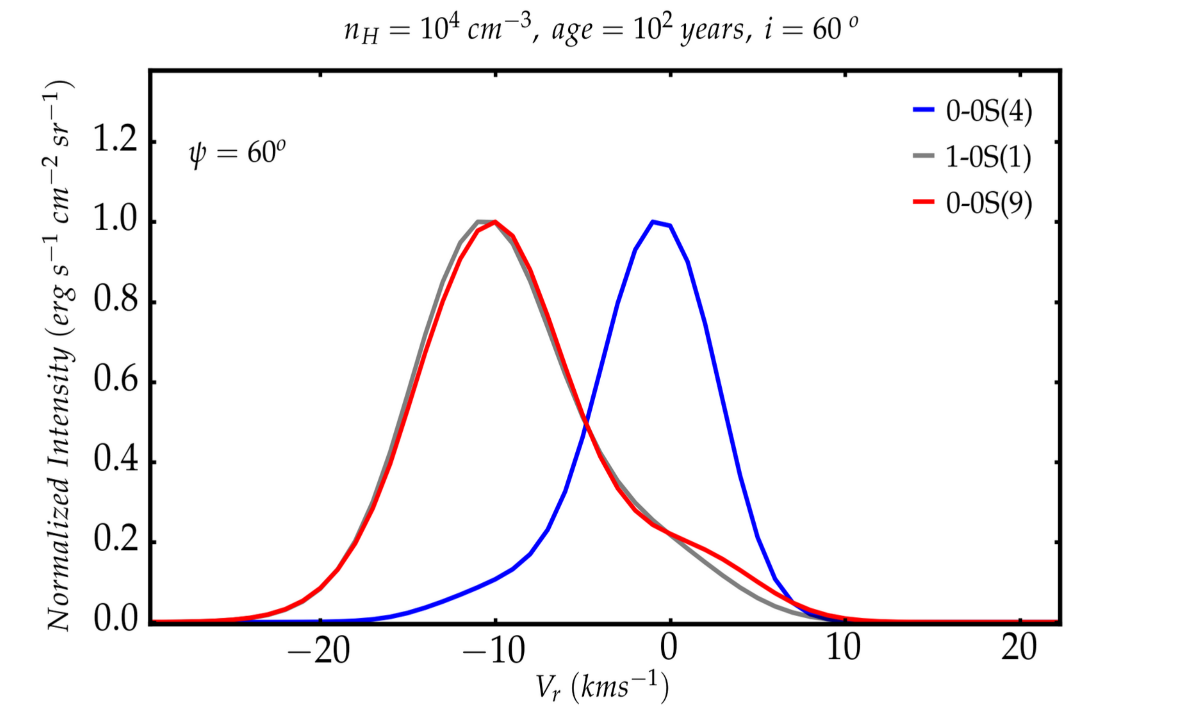


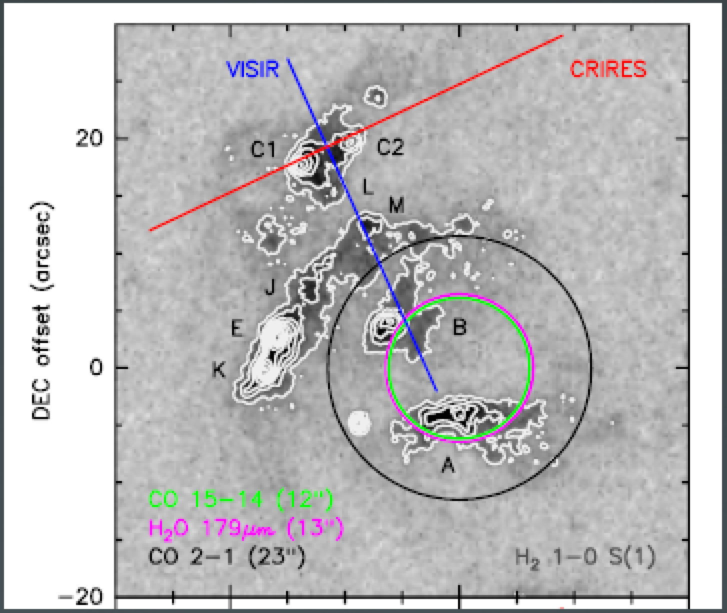
Map: CO(6-5)
Contours:
CO(3-2)

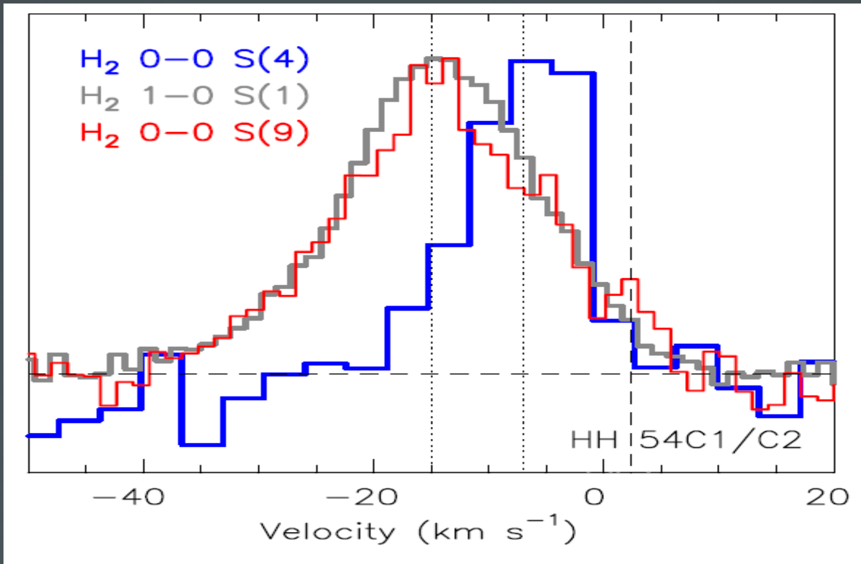
Map: 8 micron
Contours:
H_2 0-0S(5)
SiO(5-4)

H_2 line shapes in HH54

Tram et al. (2018)
Line computation for a full bow shock





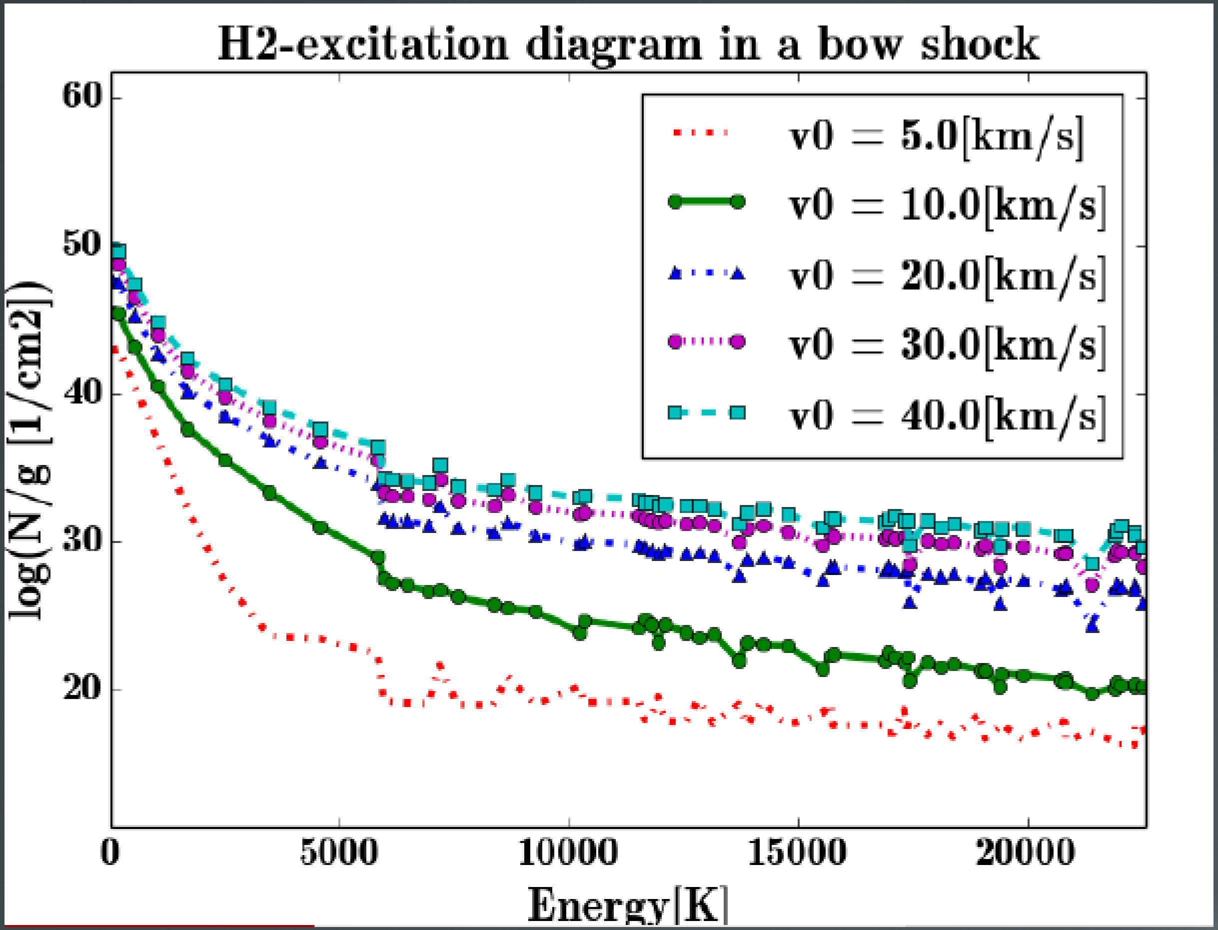


HH54 *Slit* measurements by Santangelo et al. (2015)

<u>Caveats:</u>
- **CRIRES vs VISIR: calibration ?**
- **Slit measurement vs. full bow line shape**
- **Slit position is not the same for the two instruments.**

Nevertheless: 0-0S(4) probes ambient speed (C-shocks)
the other two lines probe material at jet speed (J-shocks)
=> Genuine shift Between lines ?

Resulting H2 excitation diagram



(see Tram's PhD thesis 2018)

Less changes at higher velocities:
threshold effect and low-velocities domination
=> will improve future interpretation of shock observations.

Summary

- Compressive linear waves steepen and form shocks

=> they are very common

- Energy dissipation in shocks is mediated by viscous, resistive or ion-neutral friction
- Shocks convert irreversibly ordered energy into disordered energy (thermal → internal → radiation)
- We observe this radiation and can probe the dynamics which generated the dissipation

Thanks for your attention !

HST Image of wind blown bubble N44F

Bow shocks in the sink

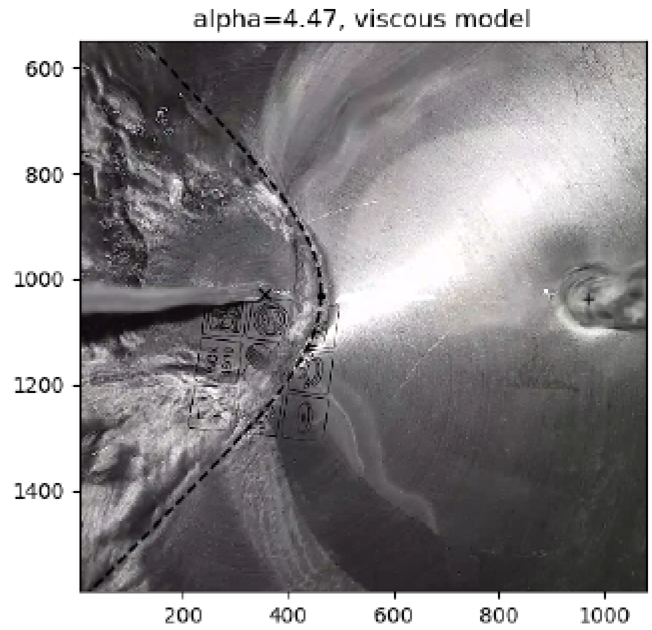


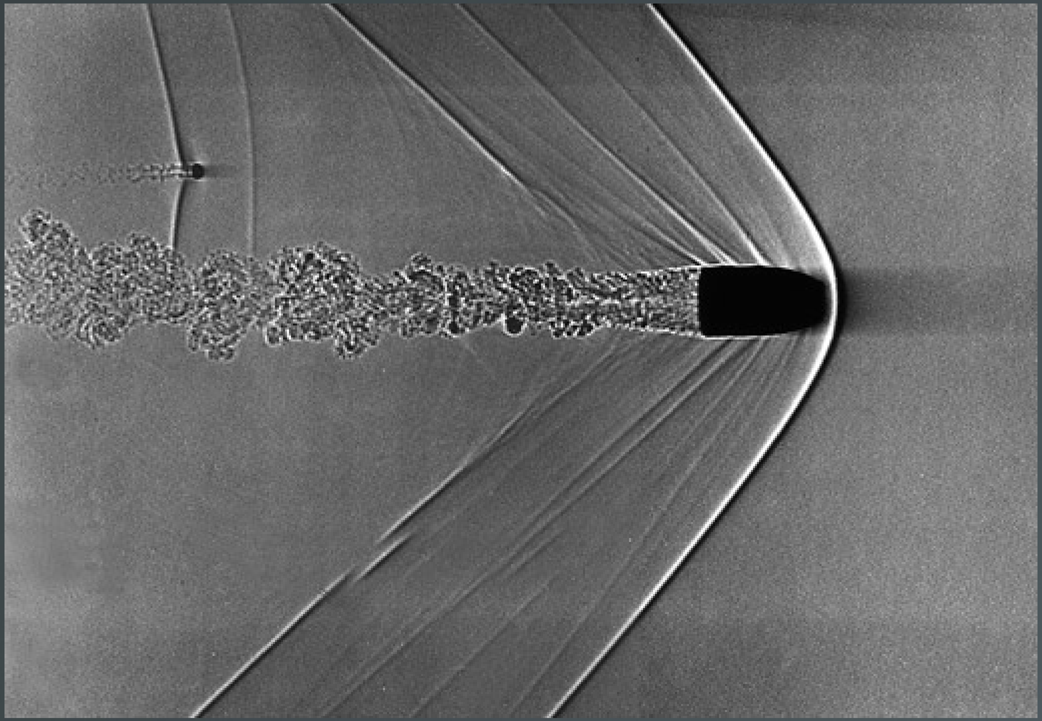
Figure 18: Intersection of two shallow water flows originating from two taps (viewed from above, '+' and 'x' signs mark the positions where both taps hit the experimental plane). The dashed line marks the position of the hydraulic jump as computed from ram pressure balance. The dimensionless parameter $\alpha = (D_1/D_2)^{3/4}$ where D_1 and D_2 are the flows at both taps can be measured from adjusting the theoretical curve by eye ($\alpha = 5 \pm 1$) or by measuring the flows at the two taps ($\alpha = 7 \pm 1.6$ for $D_1 = 200$ mL.s^{-1} and $D_2 = 15$ mL.s^{-1}, assuming a 30% relative uncertainty while measuring the flows' ratio). Experiment conducted with Merwan Ouldelhkim during his internship at ENS in may 2017.

Shocks are everywhere

R. Doisneau ("Caniveau en crue")

Flow around an obstacle

Shocks: bullet



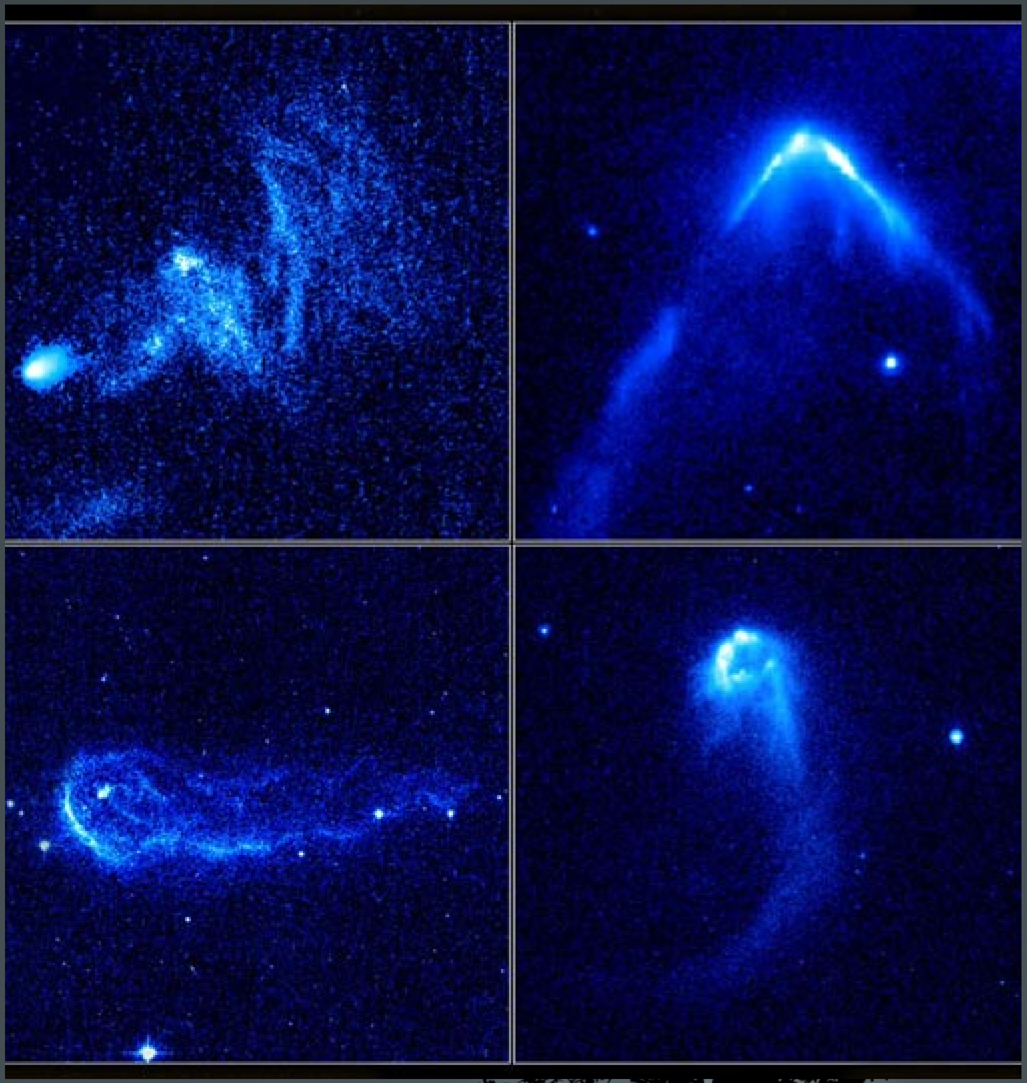
Shocks: cannon


(NavSource Naval History)

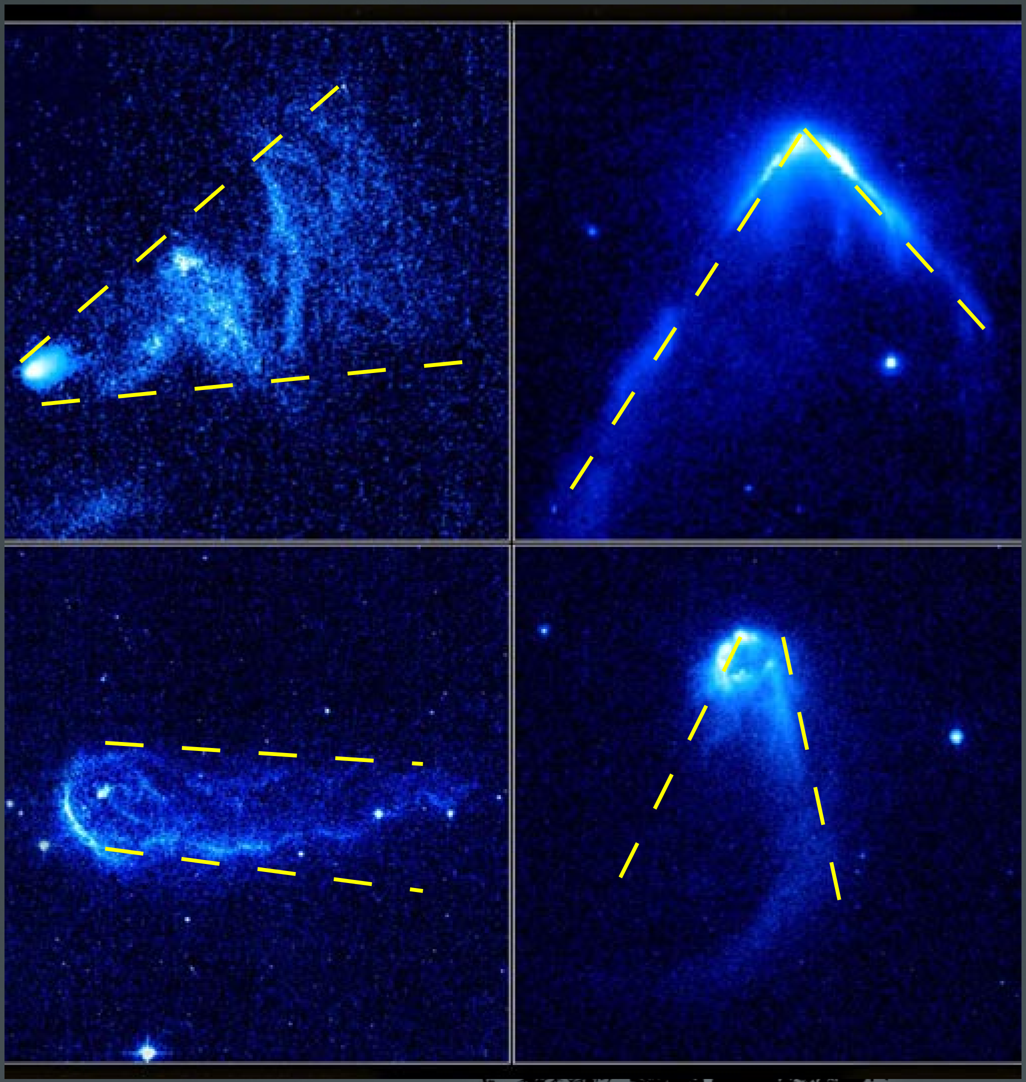
Supersonic car

Shocks: supersonic plane

Shocks: runaway stars



Shocks: runaway stars

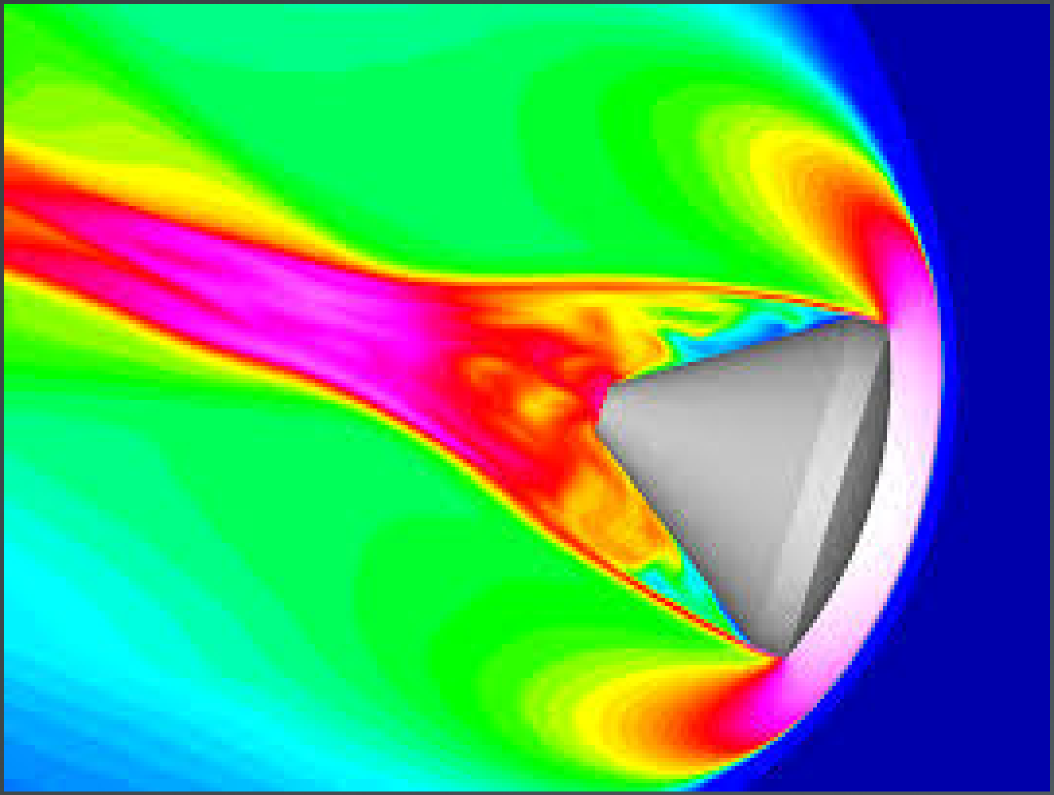


Shock: Galaxy cluster collisions

Shock: Galaxy cluster collisions

Wakes usually are waves, not shocks

Atmospheric re-entry



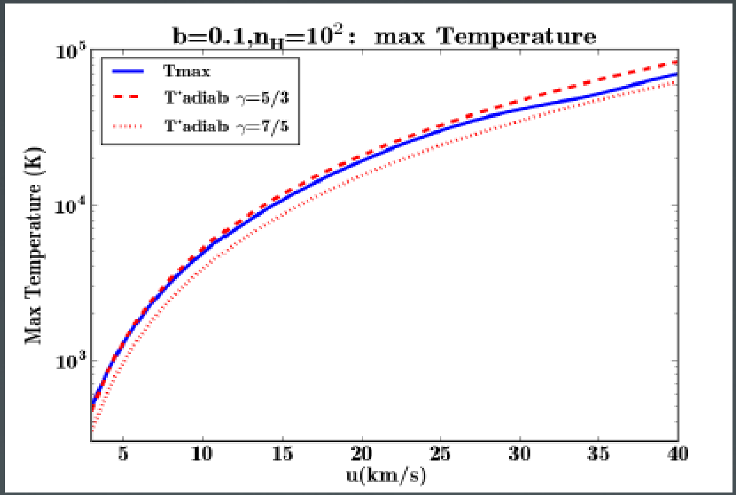
Credits: A. Reagan, Vermont university

Atmospheric re-entry the heating problem

- Energy conservation at the shock surface:

For the molecular weight of the ISM:

$$T_{\rm max} = 53\,{\rm K}\,(u/1\,{\rm km\ s^{-1}})^2$$

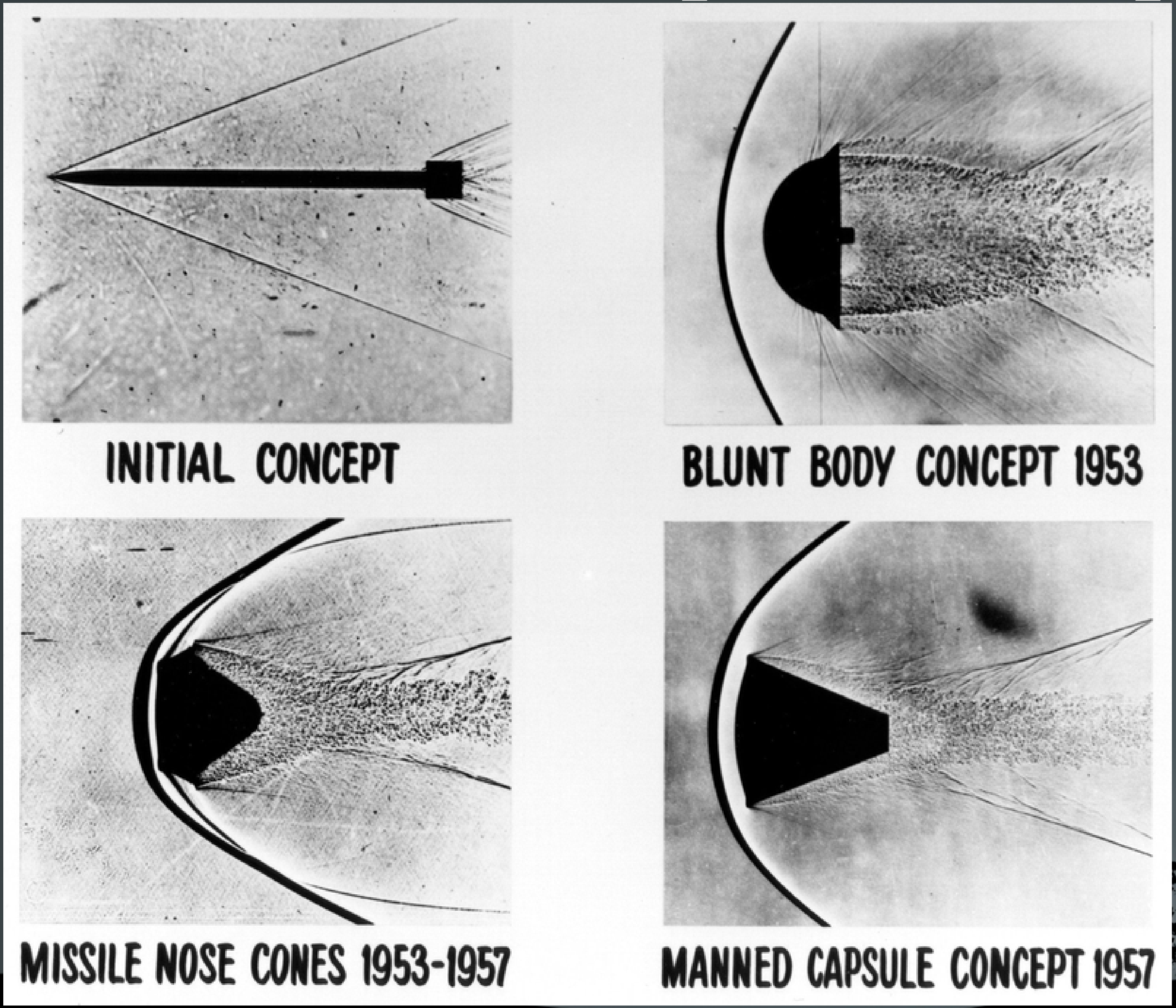


Lesaffre et al. 2013

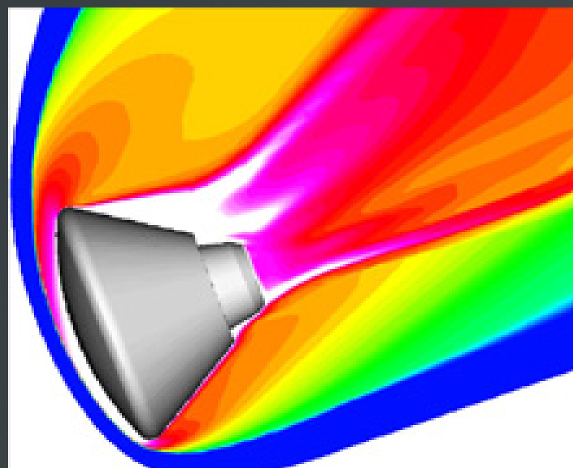
- Escape velocity: 11.2 km/s
- Melting temperature of steel: ~1700 K

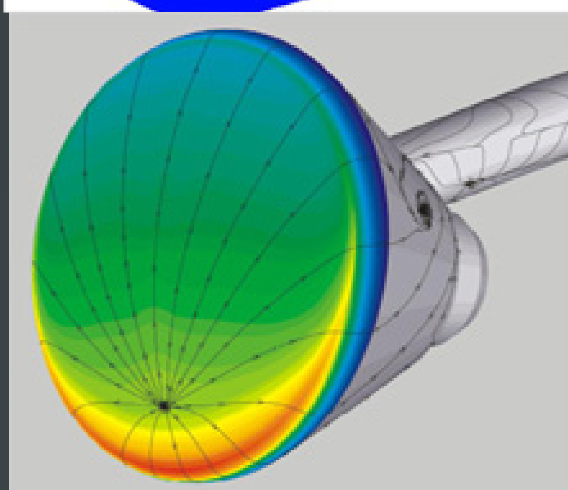
“At such speeds, probably even in the thinnest of air, the surface would be heated beyond the temperature endurable by any known material. This problem of the temperature barrier is much more formidable than the problem of the sonic Barrier.” ***Theodore von Kármán***, 'history of aeronautics', 1954

Atmospheric re-entry: the search for the perfect shape



Atmospheric re-entry



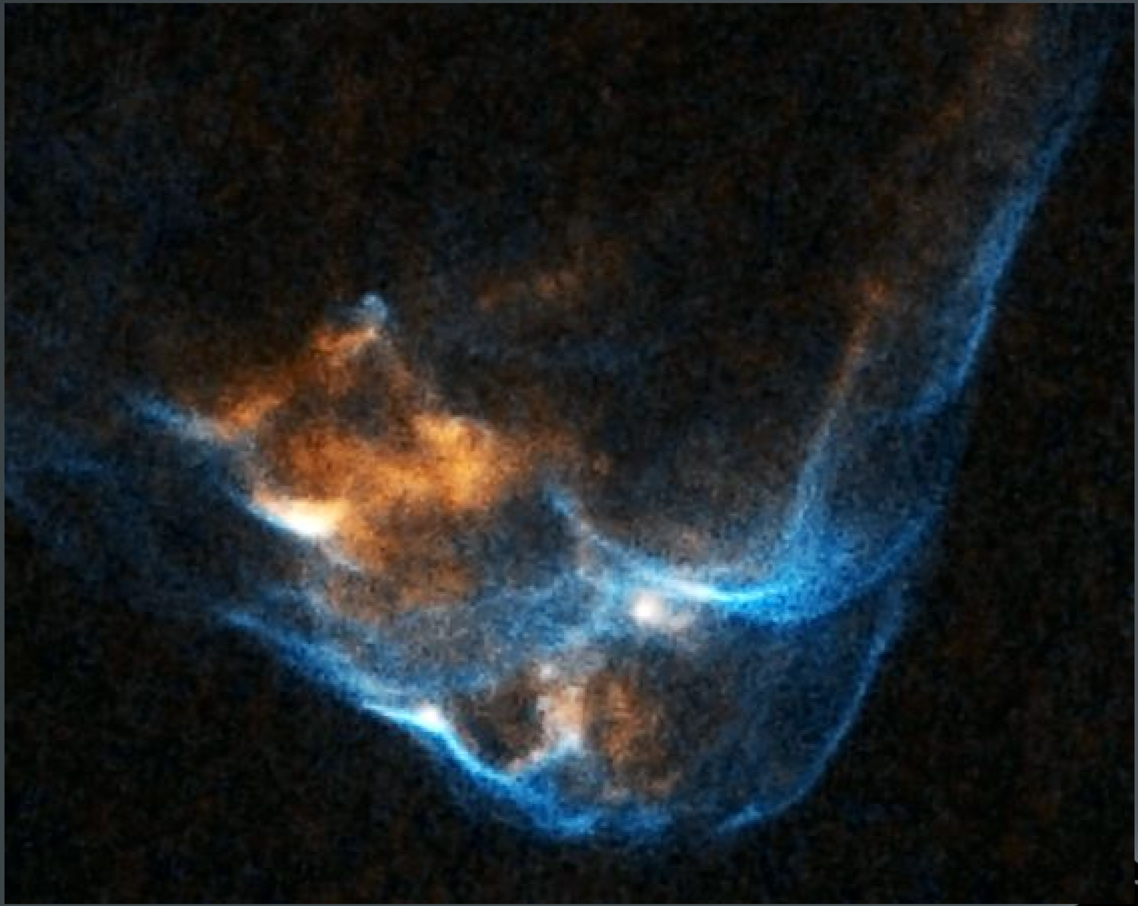


Credits: German Aerospace Center

Atmospheric re-entry: imprint on meteorites

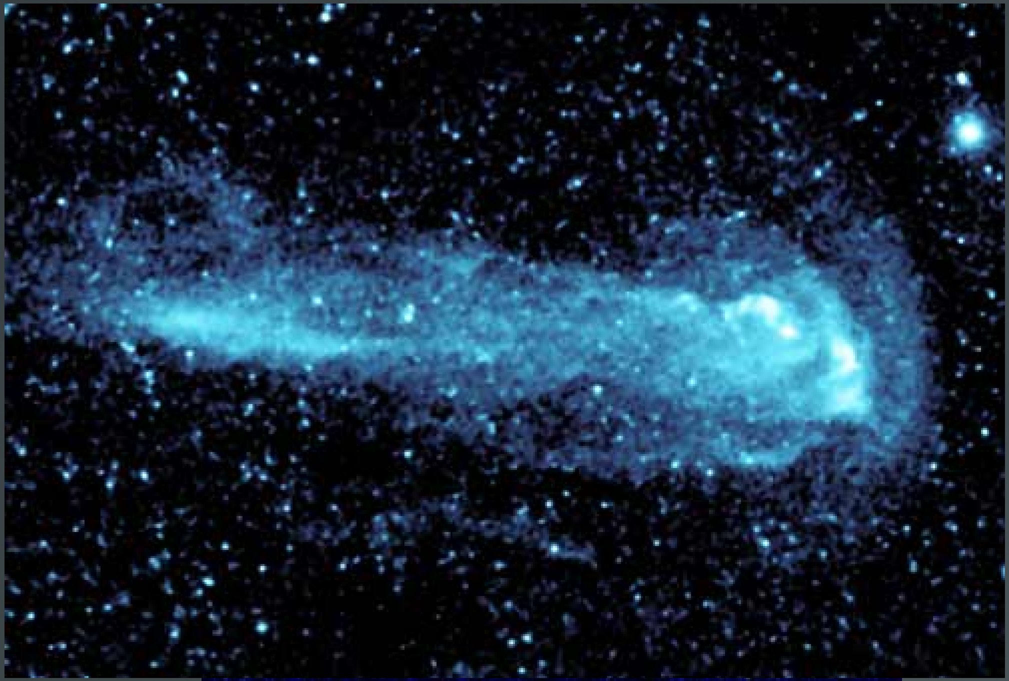
Betelgeuse

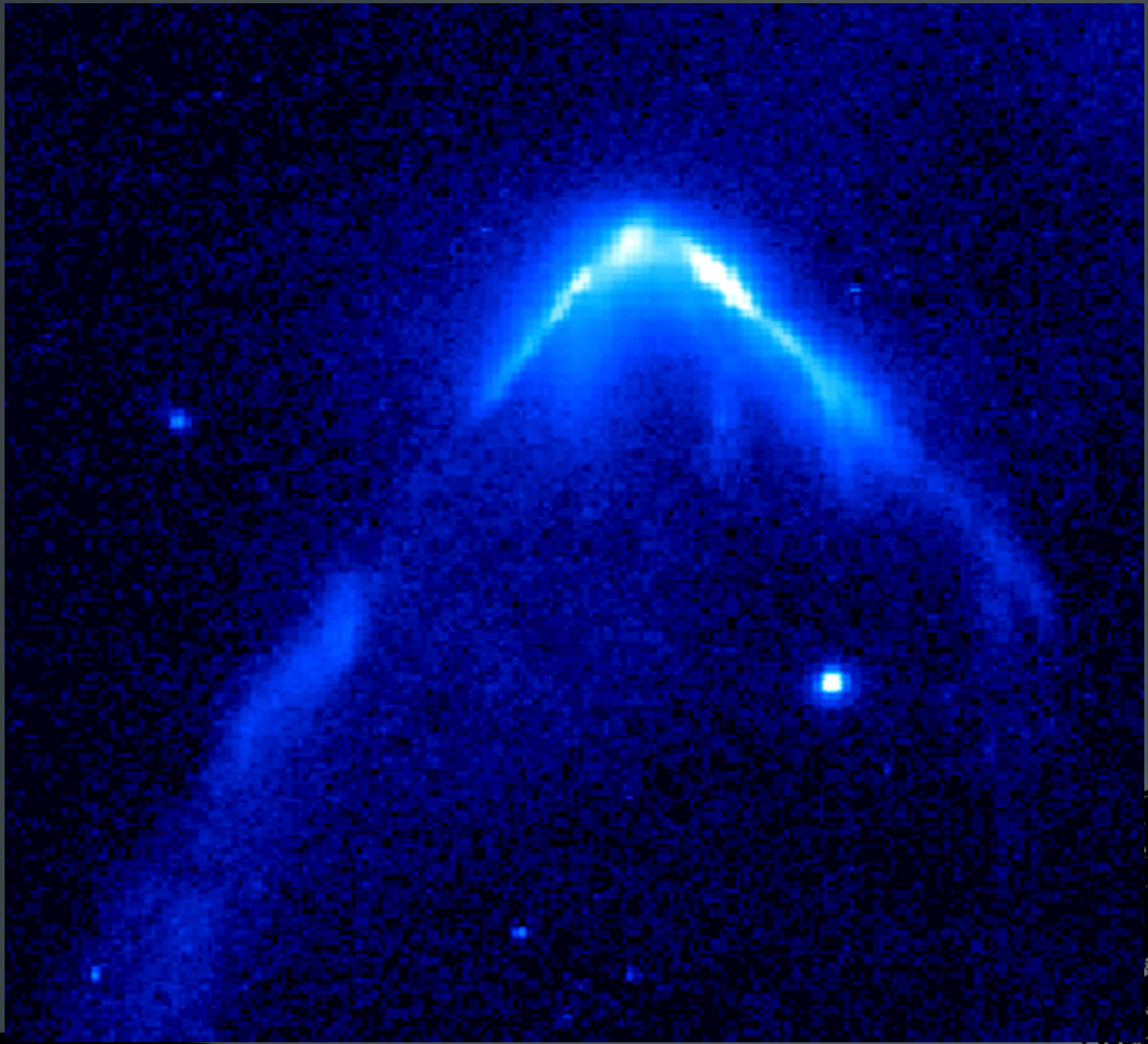
HH 34 close-up



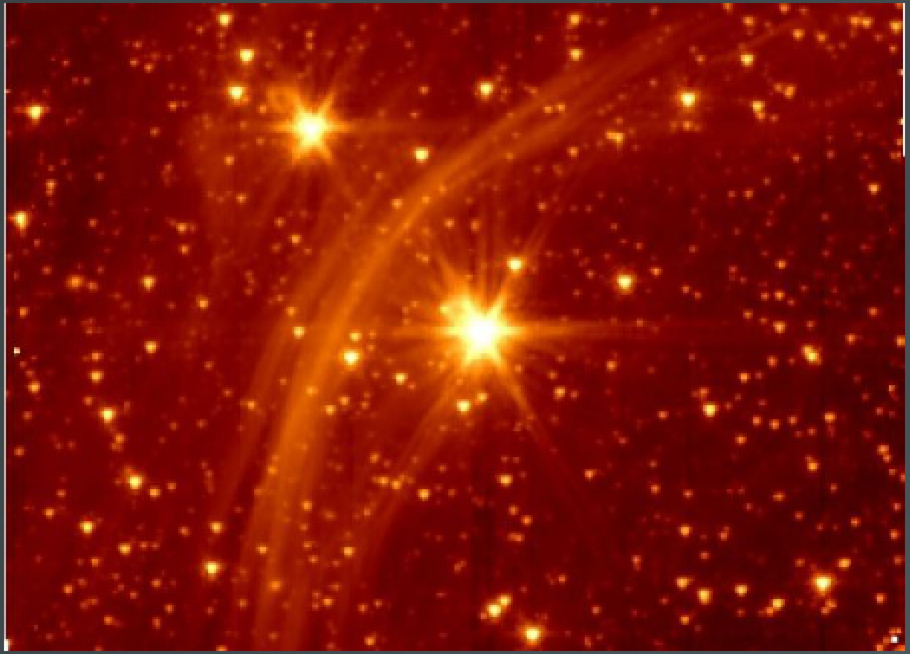
BZ Cam

Mira

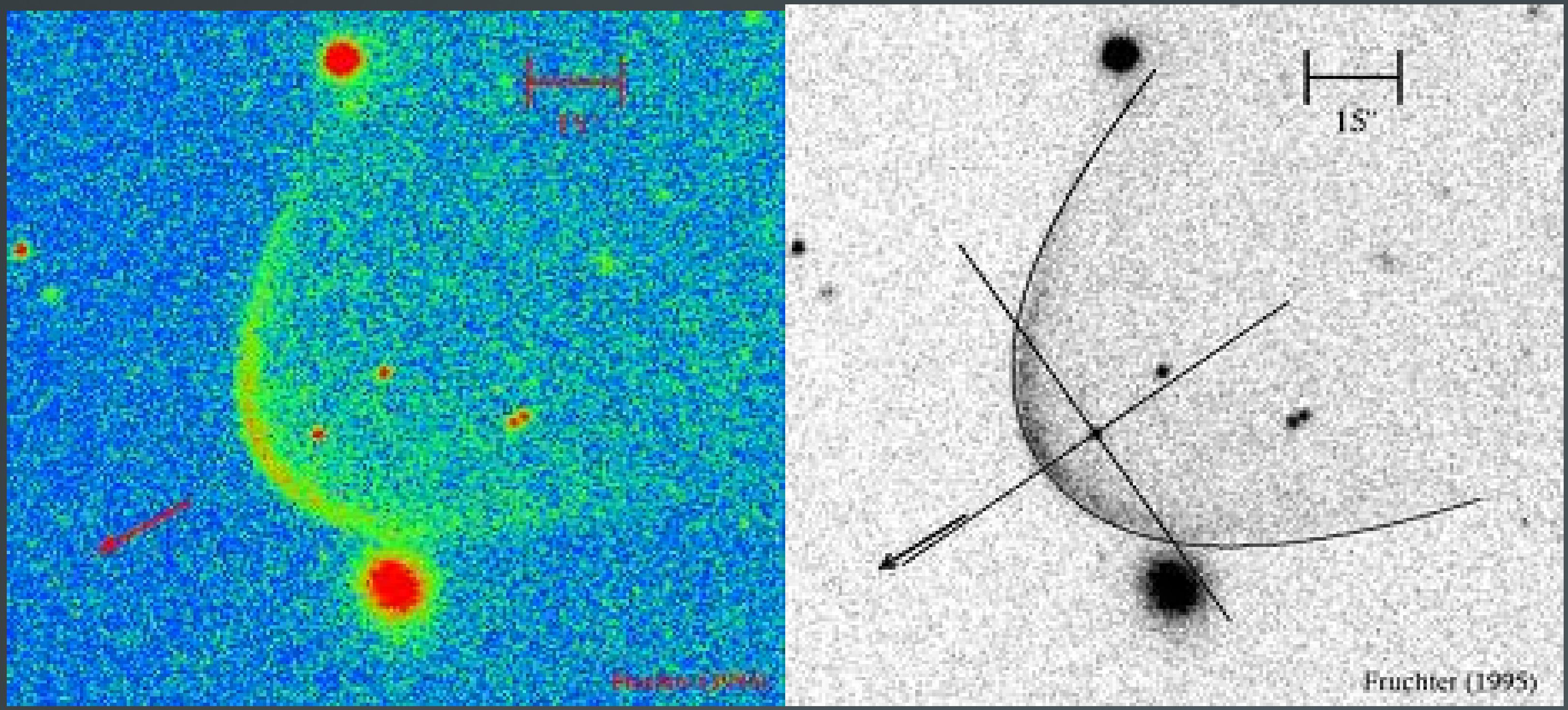




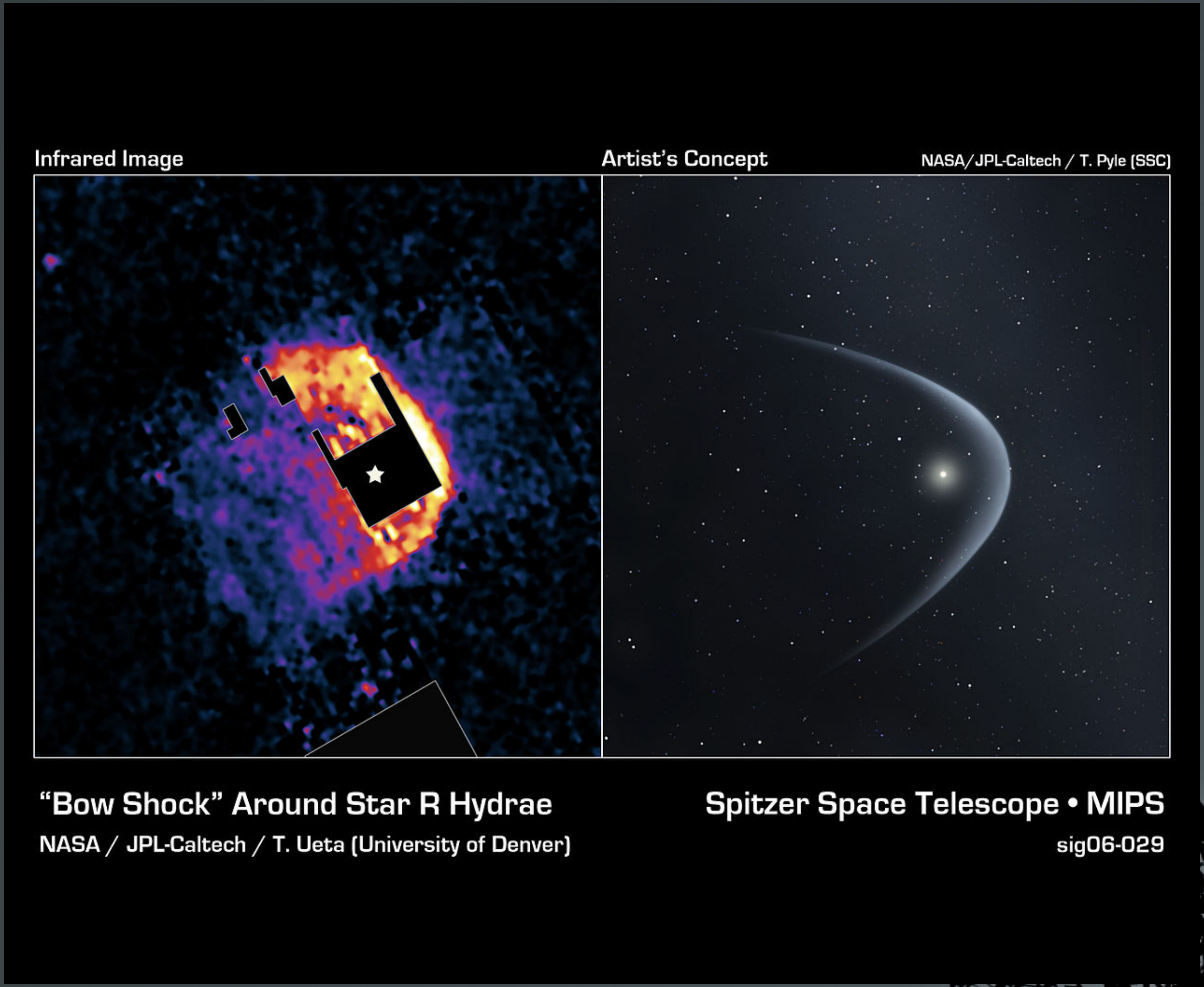
Vela X



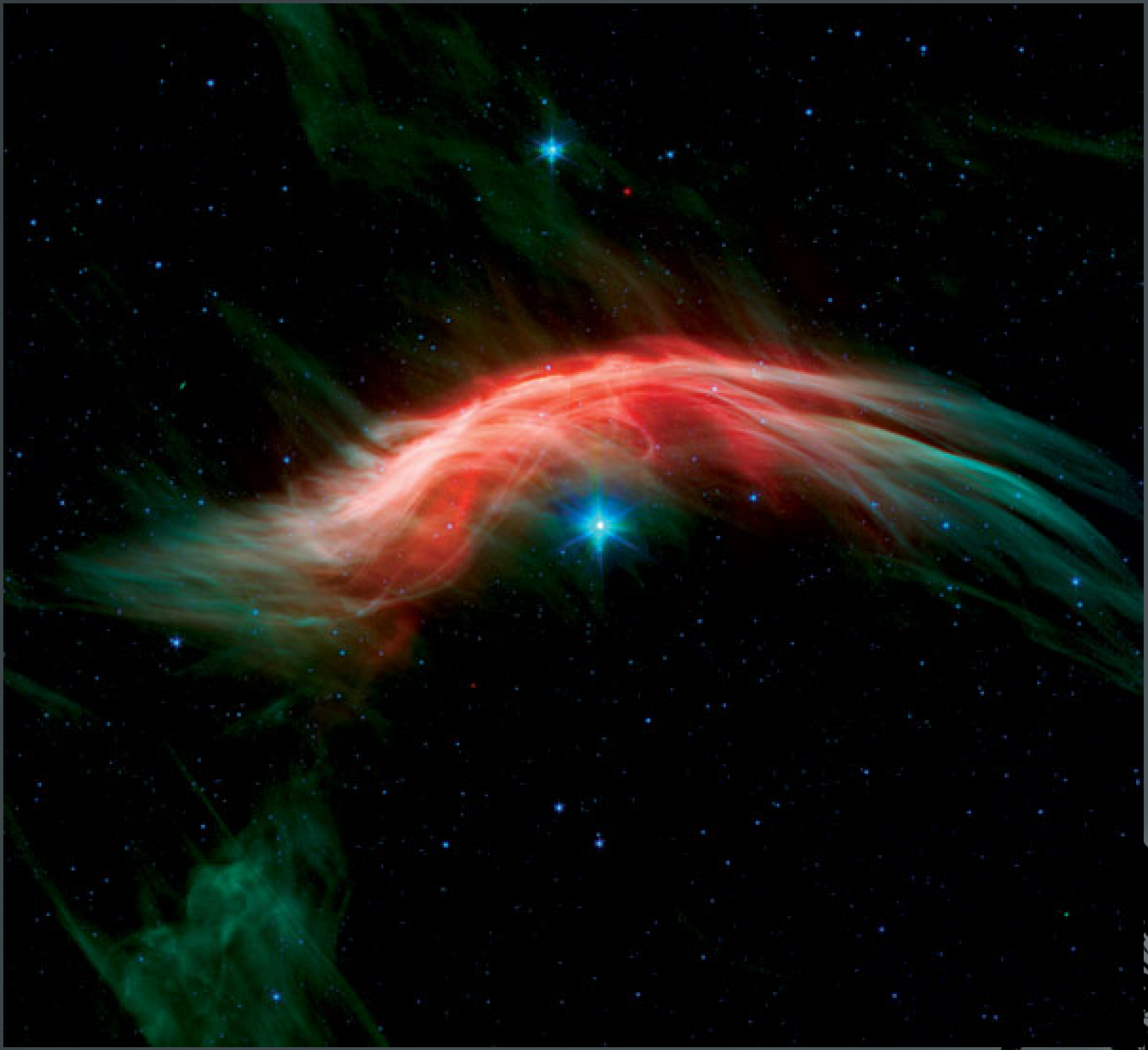
PSR J0437-4715



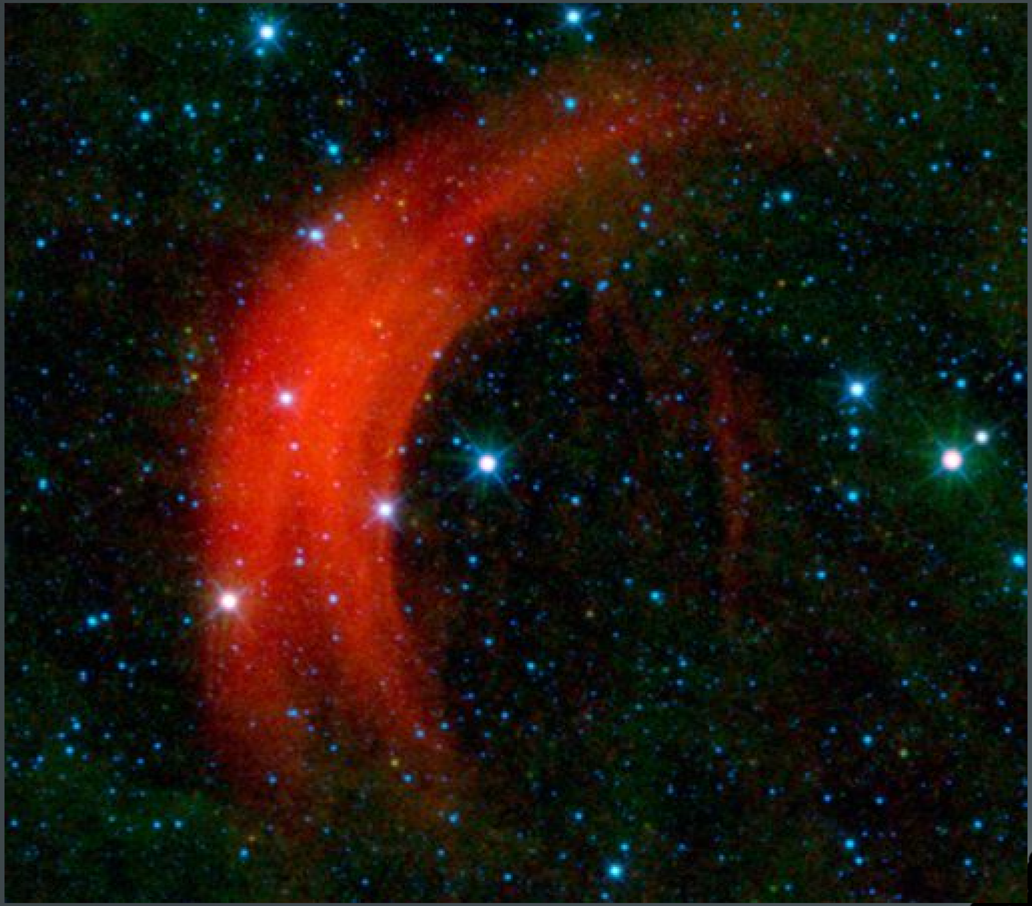
RR Hydrae


Infrared Image
Artist's Concept
NASA/JPL-Caltech / T. Pyle (SSC)
"Bow Shock" Around Star R Hydrae
NASA / JPL-Caltech / T. Ueta (University of Denver)
Spitzer Space Telescope • MIPS
sig06-029

Zeta Oph



Wise 33155



Mach 6 turbulence simulation